2 A snapshot of a region in the context of global general equilibrium

GTEM consists of three main modules: the economic core, the population module and the environment module. The economic core describes the optimal choices made by the agents subject to the opportunities and constraints that they face. The population module explains how the structure of both the population and the labour force changes globally in response to altered economic environments and thus helps to establish a two-way feedback between the demographic system and the economic core. The environment module tracks the emissions of greenhouse gasses that result from the state and nature of economic activities and lifestyle. At the centre, therefore, lies the economic core that has two-way feedback with the population module and a one-way linkage with the environment module.

This chapter focuses on the economic core of the model. It describes the basic economic relationships that are expected to hold in any arbitrary region when that region is in general equilibrium as an integral part of the global economy. By generalising these relationships to all other regions and imposing equilibrium conditions for global agents, a snapshot is presented of the general equilibrium of the global economy that GTEM is designed to represent.

Assume now that the world is divided into r economic regions (1, 2, …, r). A region can be a part of a country, a country or a group of countries, provided data are available. In each region, there are assumed to be nine types of conceptual economic agents:

1. industries
2. primary factors
3. households
4. government
5. tax system
6. exporters
7. importers
8. investors
9. finance brokers

In addition to these nine types of agent located in each region there are another three types of agent located in 'international waters':

10. global traders
11. global transport systems
12. global finance centres (clearing houses)

Briefly, these agents can be introduced as follows. In each region (set REG), there are j industries (in set PROD_COMM), each producing single products (in set PROD_COMM) using primary factors (in set ENDW_COMM) and intermediate inputs (from the set of traded commodities TRAD_COMM), both produced locally and imported.[1] They pay taxes on all purchases and possibly for producing outputs as well.

[1] The sets defined in parentheses are for exposition only. The actual sets used and their definitions are provided at the end of this chapter.

Primary factors are owned by households, who also receive all tax revenues collected in the region and make all transfer payments to the rest of the world. All households are assumed to be identical in all respects, with homogeneous preferences. A representative household in a region therefore is viewed as an ‘average resident’ of the society who receives the per person income of the region. As a society, households together allocate their income into consumption of private goods, public goods (by funding government expenditure fully) and savings.

The government provides public goods. It uses the money made available by regional households to purchase goods and services, both produced locally and imported, as inputs to the production of ‘public goods’ of the same value. The tax system imposes tax on every commodity transaction and transfers the proceeds to households as a lump sum. Exporters purchase commodities from domestic industries at market prices, pay export tax to the tax system and sell the goods to global traders. Importers buy goods produced overseas from global traders, pay import duty to the local government and sell imported commodities to various domestic users at market prices.

Investors in each region decide, independently of the savers, how much to invest in the region over the given period, taking into account the rates of return that they expect on their investment. They finance their investment fully by issuing bonds. Finance brokers in each region buy bonds from local investors, sell them to local households as demanded, and sell the rest in the world market. They clear their bond market by trading with the global finance centre, where bonds issued in various regions are considered perfect substitutes.

The global finance centre allows regional financial brokers to buy and sell bonds globally on behalf of regional households and regional investors. In addition, the global finance centre facilitates the servicing of foreign debt by regions and transfers the proceeds to the creditor regions.

Global traders purchase goods from the exporters of the exporting regions and sell to the importers of the importing regions. In delivering the goods from exporters to importers, traders use the transport services provided by the global transport sector.

Given that these twelve types of agent are involved in some sort of transactions with each other, their transactions with each other can be viewed in a matrix, which can be called a global social accounting matrix (SAM). A global SAM would be a very convenient tool for obtaining a snapshot of the global economy in equilibrium as it traces all sources of income and all items of expenditure, exhausting the incomes of all agents in all regions simultaneously. As a global SAM would be complex to present, as well as to navigate, the focus is the SAM of an arbitrary region and its transaction with global agents. With a bit of a stretch of imagination, it is not difficult to see how the global SAM would look and how the regional agents interact with each other through the global agents. This discussion will lead to the derivation of some of the conditions that would be expected to hold at the general equilibrium of the global economy.

Table 2.1, which provides a stylised SAM of an arbitrary region of the global economy, can be navigated using the naming conventions introduced in box 2.1. The table contains twelve numbered columns and twelve numbered rows for the twelve agents described above whose names are given in the unnumbered second column and the unnumbered second row of the table.

Table 2.1 **A stylised social accounting matrix (SAM) of a region in GTEM**

	To	(1) Inds.	(2) Factors	(3) H'holds	(4) Govt.	(5) Taxes	(6) Exporter	(7) Importer	(8) Investor	(9) Broker	**(10) Global Trader**	**(11) Global Trans.**	**(12) Global Finance**	**Total Income**
(1)	Inds.	VDFM		VDPM	VDGM		VXMD		INV			**VST**		**VOM**
(2)	Factors	EVFM												**EVFM**
(3)	H'holds		EVOA			Total tax rev.							**- FY**	**GNP**
(4)	Govt.			VGA										**VGA**
(5)	Taxes	(VDFA-VDFM)+(VIFA-VIFM)+(EVFA-EVFM)+(VOM-VOA)	EVFM-EVOA	(VDPA-VDPM) + (VIPA-VIPM)	(VDGA-VDGM) + (VIGA-VIGM)		VXWD-VXMD	VIMS-VIWS						**Total tax revenue**
(6)	Exporter										**VXWD**			**VXWD**
(7)	Importer	VIFM		VIPM	VIGM									**VIMS**
(8)	Investor									INV				**INV**
(9)	Broker			SAVE									**Max {[INV-SAVE],0}**	**INV or SAVE**
(10)	**Global Trader**							**VIWS**						**VIWS**
(11)	**Global Transport**										**VTWR**			**VTWR**
(12)	**Global Finance**									**Max {0, [SAVE - INV]}**				**SAVE - INV**
	Total Exp.	**VOM**	**EVFM**	**GNP**	**VGA**	**All tax rev.**	**VXWD**	**VIMS**	**INV**	**INV or SAVE**	**VIWS**	**VST**	**[INV–SAVE] + FY**	

Box 2.1: **Naming conventions for elementary data matrices in GTEM SAM**

Following GTAP, a naming convention has been adopted in GTEM to represent the elementary data — the data that are read. Most elementary value coefficients are given four letter names. These letters, with some exceptions, as listed below, are chosen to identify the nature of the flows.

The **first letter** is either *E* or *V*. *E* indicates that the value is associated with an endowment commodity (primary factor) and V indicates that the value is of any other type.

The **second letter** is either *V* or *D* or *I*. *V* denotes that the value refers to endowment commodities, which are considered nontradable commodities, and *D* and *I* denote the sources of traded commodities — *D* for domestic and *I* for imports.

The **third letter** in the coefficient name represents the type of the agent involved — *O* for the seller or producer of the output, and for buyers: *P* for private households, *G* for government and *F* for industries (firms).

The **fourth letter** is reserved for the type of price employed to value the flow — *A* for agent's price (and the agent could be a buyer or the seller as the case may be), *M* for market price and *W* for world price (distinction between fob and cif prices will be made later on).

Following this convention, the various values of the flows occurring in the input–output part of the SAM, with their corresponding notations, are described as follows:

VDFM(i,j,r) value of domestic commodity *i* purchased by industry *j* in region *r* at market prices

VDFA(i,j,r) value of domestic commodity *i* purchased by industry *j* in region *r* at agents' prices (i.e. at prices paid by firms)

VIFM(i,j,r) value of imported commodity *i* purchased by industry *j* in region *r* at market prices

VIFA(i,j,r) value of imported commodity *i* purchased by industry *j* in region *r* at agents' prices (i.e. at prices paid by firms)

EVFM(i,j,r) value of endowment commodity (primary factor) *i* purchased by industry *j* in region *r* at market prices

EVFA(i,j,r) value of endowment commodity (primary factor) *i* purchased by industry *j* in region *r* at agents' prices (i.e. at prices paid by firms)

EVOA(i,r) value of output of endowment commodity (primary factor) *i* supplied by the household in region *r* at agents' prices (i.e. at net of tax prices received by the seller)

VDPM(i,r) value of domestic commodity *i* purchased by private households in region *r* at market prices

VDPA(i,r) value of domestic commodity *i* purchased by private households in region *r* at agents' prices (i.e. at prices paid by households)

VIPM(i,r) value of imported commodity *i* purchased by private households in region *r* at market prices

VIPA(i,r) value of imported commodity i purchased by private households in region *r* at agents' prices (i.e. at prices paid by households)

VDGM(i,r) value of domestic commodity *i* purchased by the government in region *r* at market prices

VDGA(i,r) value of domestic commodity *i* purchased by the government in region *r* at agents' prices (i.e. at prices paid by households)

VIGM(i,r) value of imported commodity *i* purchased by the government in region *r* at market prices

VIGA(i,r) value of imported commodity *i* purchased by the government in region *r* at agents' prices (i.e. at prices paid by households)

Exceptions include trade matrices:

VXMD(i,s,r) Value of exports of commodity *i* from region *s* to destination region *r* at domestic market price of the source region s

VXWD(i,s,r) Value of exports of commodity *i* from region s to region *r* at world (fob) price

VIWS(i,s,r) Value of imports of commodity *i* from region *s* into region *r* at world (cif) price

VIMS(i,s,r) Value of imports of commodity *i* from region *s* into region *r* at duty-paid market price of the importing region *r*

VST(m,r) Value of sales of margin commodity *m* made to the international transport sector at domestic market prices of the source region *r*

VTWR(m,i,s,r) Value of margin commodity *m* used to deliver exports of commodity *i* from region *s* to region *r* at world price

SAVE Savings of regional households

FY Net transfer payments to foreign countries, including interest payments.

In the following sections, where relevant, other data coefficients that are derived from these elementary data matrices are introduced and new ones added as required.

The agent named in the numbered first column of table 2.1 and the agent named in the numbered first row of the table match exactly, and so on. Hence, for each agent there is a row and a column in the SAM. The entries along the row show the incomes received by the agent from the nine regional and three global agents. Similarly, the entries along the corresponding column show the payments made by the agent to the nine local agents and the three global agents. The last row sums the expenditure and the last column sums the income of the corresponding agent.

A necessary condition for equilibrium is that all agents satisfy their respective 'budget' constraints (or zero-profit conditions). In other words, it is necessary in a SAM that the total income of each agent be equal to its total expenditure.

Before examining the transactions covered in the SAM, it is useful to take note of the way that tax revenues are represented in the matrix. Generally, the difference between the flows valued at agents' prices (that is, the flows with names ending with *A*) and the corresponding flows valued at market prices (flows with names ending with *M*) yield the tax revenues received or the subsidies paid on the respective transaction. For example, *VDFA(i,j,r)* – *VDFM(i,j,r)* = taxes paid by industry *j* in region *r* on the purchase of input *i* from the domestic source.

There are two exceptions to this rule, however:

(i) $$VOM(j,r) - VOA(j,r) = PTAX(j,r),$$

which means that the agent's price plus the production tax gives the market price value of sectoral output, and

(ii) $\sum_j EVFM(i,j,r) - EVOA(i,r) =$ income taxes paid,

which means that the take home pay of factors, *EVOA*, equals the payments made to factors by firms, which is *EVFM*, less the income taxes paid.

Going through agent by agent, in turn, shows how agents transact with each other. Take the industries column first: industries spend on purchasing inputs from domestic sources, which is given by *VDFM*; they make payments to the factors employed, which is *EVFM*; they make payments to importers for buying imported intermediate inputs, which is *VIFM*; and pay taxes to the tax system on all of these transactions as well as paying production taxes (or receiving subsidies).

For each sector *j* in region *r*, summing the corresponding tax inclusive payments for the purchase of inputs yields:

$$VOA(j,r) = \sum_{i \in COM} [VDFA(i,j,r) + VIFA(i,j,r)] + \sum_{k \in FAC} EVFA(k,j,r) \tag{2.1}$$

where *VOA* measures the total cost of producing output, excluding the production taxes. Clearly, to have a zero-profit condition satisfied, producers must receive *VOA* as the value of output net of production taxes.

Similarly, taking the production taxes/subsidies, *PTAX(j,r)*, into account explicitly, the total cost of production of each sector *j* in region r can be expressed as:

$$\begin{aligned} TC(j,r) = &\sum_{i \in COM} [VDFA(i,j,r) + VIFA(i,j,r)] \\ &+ \sum_{k \in FAC} EVFA(k,j,r) + PTAX(j,r) \end{aligned} \tag{2.2}$$

where *TC(j,r)* represents the total cost of industry *j* in region *r*.

Going through the industry row, it is clear that industries sell some of their outputs to the industries themselves for intermediate use at market price and value *VDFM*. They also sell to households, worth *VDPM*, to the government, worth *VDGM*, to exporters of the goods, worth *VXMD*, to investors the capital goods, worth *INV*, and margin goods, worth *VST*, to the global transport sector.

Total sales revenue at market price, which must accrue to the producer, can be summed as:

$$\begin{aligned} VOM(i,r) = &\sum_{j \in IND} VDFM(i,j,r) + VDPM(i,r) + VDGM(i,r) \\ &+ \sum_{s \in REG} VXMD(i,r,s) + \delta_i \, VST(i,r) \end{aligned} \tag{2.3}$$

where $$\delta_i = \begin{cases} 1 & \text{if i is a margin cmmodity} \\ 0 & \text{if i is not a margin commodity} \end{cases}$$

and *VOM* represents the value of output at market price, which is the same for all agents.

Clearly, the zero-profit condition implies that *VOM(j,r) = TC(j,r)* for each industry *j* and region *r*.

Row 2 also shows that, even though it costs *EVFA* for industries to employ the primary factors, the factors actually receive *EVFM*, with the difference paid to the tax system as factor employment tax, such as payroll tax. From the corresponding column 2, factors use this income to pay *EVOA* to households (the owners) and the rest as source specific income tax to the tax system. Hence, households receive *EVOA* as after-tax factor income.

The zero-profit condition for each factor can be written as:

$$\sum_{j \in IND} EVFM(i,j,r) = EVOA(i,r) + \text{factor-income tax revenue(i,r)}; \tag{2.4}$$

for all $i \in FAC$.

Now take row 3, which shows that households receive *EVOA* as factor income, all tax revenues from the tax system, and make net transfer payments (including net interest payments) to foreigners, *FY*. The sum is the gross national product (GNP) of the region. Regional households together — as a superhousehold — allocate the GNP into private consumption, public consumption (provision for government expenditure), *VGA*, and savings, *SAVE*. In purchasing goods for private consumption expenditure, households pay *VDPM* to domestic industries for domestic goods and *VIPM* to importers for imported goods and pay taxes to the tax system on all purchases. The tax inclusive costs of purchases are *VDPA* for domestic goods and *VIPA* for imports.

As households satisfy their respective budget constraints, for any region *r*, then:

$$\begin{aligned} &\sum_{i \in FAC} EVOA(i,r) + \textit{All net tax revenues} - FY(r) \\ &= \sum_{i \in FAC} \sum_{j \in IND} EVFM(i,j,r) + \textit{All net indirect tax revenues} - FY(r) \\ &= GNP(r) \\ &= \sum_{i \in COM} [VDPA(i,r) + VIPA(i,r)] + VGA(r) + SAVE(r). \end{aligned} \tag{2.5}$$

The left hand side of the equation (2.5) expresses GNP of a region in terms of after (income) tax factor incomes, *EVOA*, and total tax revenue and international transfer payments. The second part expresses regional GNP in terms of standard accounting concepts — as a sum of the value added at market price, indirect tax revenue and net international transfers. Whichever method is preferred, equation (2.5) shows the aggregate budget constraint of the regional households. In the SAM presented here, the left most part of the equation has been adopted to derive the aggregate household income of the regions.

In row 4, the government receives income from households, given by *VGA*. In column 4, it can be seen that the government spends the money it has received to pay for its consumption. It pays *VDGM* to domestic industries for domestic commodities and *VIGM* to importers for the imported goods it consumes and pays taxes on its purchases, if any. The tax inclusive expenditures of the government satisfy the government's budget constraint:

$$VGA(r) = \sum_{i \in COM} [VDGA(i,r) + VIGA(i,r)] \quad \text{for all } r \in REG. \tag{2.6}$$

Row 5 and column 5 of the table are straightforward. They indicate that the tax system collects tax on all transaction and hands over the proceeds to households in a lump sum.

Column 6 shows that exporters pay *VXMD* to domestic industries for purchasing the commodities that they export, and pay export tax to the region's tax system. The tax inclusive value of the payments made by the exporters is *VXWD*. Row 6 shows that exporters receive their income, *VXWD*, from global traders who purchase the exports at fob prices, which is inclusive of export taxes. Thus the zero-profit condition of the exporter becomes:

$$VXWD(i,r,s) = VXMD(i,r,s) + \text{export tax(i,r,s)}, \tag{2.7}$$

for each $i \in COM$, $r \in REG$ and $s \in REG$.

Row 7 and column 7 account for the incomes and expenditures of importers. The row entries indicate that importers receive *VIFM* from industries, *VIPM* from the goods they sell to households and *VIGM* from the government. The entries in column 7 show that importers pay *VIWS* to global trader for imports at cif prices, and pay import duty to the region's tax system. The duty inclusive payments made by importers are thus *VIMS*, which is reported in the last row of column 7.

The zero-profit condition for the importer therefore implies that:

$$VIMS(i,s,r) = VIWS(i,s,r) + duty(i,s,r), \tag{2.8}$$

for each $i \in COM$, $s \in REG$ and $r \in REG$.

Row 8 and column 8 show that regional investors receive income from local finance brokers, which is equal to *INV(r)*; the same amount is spent to purchase investment goods produced by the sector producing capital goods. Hence, the zero profit in investing implies that the quantity of bonds issued, *INV(r)*, is equal to expenditure on capital goods, which should be equal to *INV(r)*.

Row 9 shows the sources of income of financial brokers in the regions. They receive savings, *SAVE*, from households as payment for bonds purchased and money, equal to the maximum of zero and the excess of investment over savings, *INV – SAVE*, from the global finance centre for the bonds they sell to the world on behalf of investors. The sum of the two sources of income is equal to *INV* if the regional investment expenditure exceeds the regional savings; otherwise it will be equal to *SAVE*, the amount of regional savings.

The entries in column 9 show that the finance broker pays the amount *INV* to investors in exchange for the bonds issued by them. It also pays the maximum of zero and (*SAVE – INV*) to the global finance centre for bonds issued elsewhere, which is purchased by local households.

Thus, the finance broker earning zero profit implies that:

$$SAVE(r) + \max\{0, [INV(r) - SAVE(r)]\} = INV(r) + \max\{0, [SAVE(r) - INV(r)]\}. \tag{2.9}$$

Since, for a net saver $\max\{0, [INV(r) - SAVE(r)]\} = 0$, and $\max\{0, [SAVE(r) - INV(r)]\} = (SAVE(r) - INV(r))$ hence it follows from equation (2.9) that the income and the expenditure of the broker in a region with positive excess savings will both be equal to *SAVE(r)*. Similarly, for a region that invests more than it saves, the income and expenditure of the broker will both be equal to *INV(r)*.

The central idea used in GTEM to stylise the global financial system is based on the assumption that regional investors fund their investment by issuing bonds, which are sold to the finance broker. Households, on the other hand, save some money from their current income which they spend on bonds. It is implicitly assumed that households in each region treat domestic bonds as perfect substitutes for foreign bonds. Hence, it does not really matter to a household whether it is holding local or global bonds. Therefore, if there is a discrepancy between domestic savings and investment, the finance broker clears the bond market by accessing the global finance centre. The broker in each region sells all excess bonds, which is equal to *INV(r) – SAVE(r)*, to the global finance centre. This is how foreign savings are brought in to finance local investment. Conversely, when domestic savings exceed domestic investment, the finance broker brings foreign bonds into the region to meet the excess demand for bonds (i.e. households acquire foreign assets). In this case, domestic savings end up financing investments in foreign countries. Equilibrium in the global finance centre requires that the sum of regional savings exactly matches the sum of regional investments expressed in the same currency.

Clearly, if *INV(r)* > *SAVE(r)* for some *r*, then the region accumulates foreign debt and if *INV(r)* < *SAVE(r)* then the region ends up accumulating foreign assets. Accumulation of foreign assets or debt results in an additional flow of interest income between the region and the rest of the world and mediated by the global finance centre, which is described in row12.

These nine rows and columns complete the description of the incomes and expenditures of the various agents in a given region. In this process, regional agents enter into transactions with global agents. Unlike local agents, payments made to a global agent need not equal payments received from that global agent to each individual region but will balance globally.

First, consider the global trader. The entries in column 10 show that the global trader pays *VXWD*, the value of exports of the region at fob price, to the regional exporter and pays *VTWR*, the cost of transport margin required to deliver the goods to the destination region, to the global transport sector. At the port of the destination region, the cost of the global trader becomes *VIWS*, which is the sum of the fob price and cost of transporting the merchandise from the source to the destination. The entries in row 10 show that the global trader receives exactly the amount, *VIWS*, from regional importers in return for the merchandises. Here it is important to note that the region that exports a particular commodity is not the one who buys it. Hence, the link between the matrices *VXWD* and *VIWS* should be viewed globally. The zero-profit condition in global trading implies that for each commodity *i* produced in region *s* and exported to destination region *r*, the following must apply:

(2.10) $$VIWS(i,s,r) = VXWD(i,s,r) + \sum_{m \in MARG} VTWR(m,i,s,r).$$

It is important to note, however, that the row sum, *VIWS(i,s,r)*, which is the payment made by importers of region *r* to the global trader, is not the same as the column sum *VIWS(i,r,s)*, which is the sum of exports from region *r* to all destination regions *s*, *VXWD(i,r,s)*, and the associated cost of international transport margin $\sum_{m \in MARG} VTWR(m,i,r,s)$. The row sum gives the cif value of imports of region *r*, while the column sum gives the total exports of region *r* at fob prices, by commodities, plus the cost of shipping paid to the global agent, *VTWR*. There is no reason for these two values to be equal.

Next consider the global transport sector, which is listed in row 11 and column 11. The row shows that the global transport sector receives its income, *VTWR*, from the global trader for transporting the merchandises from source regions to their respective destinations. To provide the service, the global transport sector purchases transport commodities from various regions and pays *VST* to the regions supplying the actual input of services. The zero-profit condition for this global sector implies that for each margin commodity *m*:

(2.11) $$\sum_{r \in REG} VST(m,r) = \sum_{i \in COM} \sum_{s \in REG} \sum_{d \in REG} VTWR(m,i,s,d),$$

which means that the cost of supplying the margin services of each type *m* should be equal to the revenue received from employing the service in transporting the merchandises.

Finally, the income and expenditure of the global finance centre are considered in row 12 and column 12. From the row entries it can be seen that the global finance centre receives income from regions that are currently saving in excess of the investments they are undertaking. Hence, from region *r* the global finance centre will receive the maximum of zero and *SAVE(r) – INV(r)*. It will spend the income in paying the regions that are investing more than they are saving. From the column entries, the amount received by region *r* from the global finance centre is the maximum of zero and *INV(r) – SAVE(r)*.

It is clear that the amount of money received by the global finance centre from a region is not equal to what it pays to the region and vice versa. Naturally, this inequality leads to accumulation of foreign assets and debts. Regions service their foreign debt. They pay the interest costs of the debt to the creditor region through the global finance centre. To simplify the representation of this type of transaction, only the net receipt of interest income (and other transfer incomes) has been included and is represented by *FY*, which is paid by the global finance centre to regional households. This entry will be negative for debtor regions and positive for creditors. Since by definition, as one region's payment is another's income:

$$\sum_{r \in REG} FY(r) = 0 \,, \tag{2.12}$$

the zero-profit condition for the global finance centre implies that:

$$\begin{aligned} &\sum_{r \in REG} \max\{0, [INV(r) - SAVE(r)]\} \\ &= \sum_{s \in REG} \max\{0, [SAVE(s) - INV(s)]\} \end{aligned} \tag{2.13}$$

which can be written as:

$$\sum_{r \in REG: INV(r) > SAVE(r)} [INV(r) - SAVE(r)] = \sum_{s \in REG: SAVE(s) \geq INV(s)} [SAVE(s) - INV(s)]. \tag{2.14}$$

This means that the money paid out by the global finance centre to regions with investment greater than savings should be funded by the money received from the regions that have savings greater than their investment. When this happens the supply of savings to the global pool is exactly equal to the demand for foreign funds from the global pool. This can be seen by rearranging the above equation as:

$$\begin{aligned} &\sum_{r \in REG: INV(r) > SAVE(r)} INV(r) + \sum_{r \in REG: SAVE(r) \geq INV(r)} INV(r) \\ &= \sum_{r \in REG: SAVE(r) \geq INV(r)} SAVE(r) + \sum_{r \in REG: INV(r) > SAVE(r)} SAVE(r) \end{aligned} \tag{2.15}$$

This, in other words, means:

$$\sum_{r \in REG} SAVE(r) = \sum_{r \in REG} INV(r) \tag{2.16}$$

which means that the global financial market clears when all regions are considered together.

In chapter 3, a model is developed to explain how the underlying variables that govern the flows contained in the global SAM are determined and how they change. In doing so it is assumed that all agents are rational, and optimise subject to the constraints that they face. The important constraints are the given endowments of resources, technology, taste, and government policies.

For readers needing some basics on linearisation of equations, a primer is provided in appendix 2A, following.

Appendix 2A

Reading and interpreting Tablo code

Throughout this document, the relevant portions of the Tablo code of GTEM are presented to give model details at the technical level. The main areas of Tablo presented in this documentation are either formulas or equations. The following is a description of each key Tablo input statement, with its generic syntax, and examples. It is based on Harrison and Pearson (2002).

GTEM is solved using the GEMPACK suite of programs and so its model code is written using GEMPACK syntax. GEMPACK allows the user to solve a complex nonlinear model, which can be written in linear or level form. No matter how it is written, the model will be solved in a sequence of linear solutions (similar to the method of polynomial approximations to any curve) updating the database after each linear solution. The final solution will be obtained by extrapolating from the multistep linear solutions. In this platform, a Tablo input file is the means of communicating the theory of GTEM to the software.

The Tablo language is very similar to the language of ordinary algebra. It is controlled by the following key words:

- **FILES** – used to declare files containing various data to be read or written.
- **SETs** and **SUBSETs** – used to declare the sets and their relations of the arguments of variables, coefficients and parameters.
- **VARIABLES** – used to declare the variables of model (levels or linearised).
- **COEFFICIENTS** – used to declare level values of the variables or their functions that are either read in as data or calculated via formulas.
- **READ** – used to instruct the program to initialise some coefficients by values stored in a particular file with particular headers.
- **UPDATE** statements – used to update the values of coefficients (level variables) if they change values after each solution.
- **PARAMETERS** – used to declare coefficients that do not change their values in all time periods. Coefficients that do not change their values within a period but change over periods are initialised by **Formula** (initial).
- **FORMULA** – used to specify relationship between a coefficient and other coefficients whose values have already been initialised either through read statements or by formulas.
- **EQUATION** – used to describe the relationships between the model variables.

Writing equations in linear percentage changes

All agents in GTEM are optimisers. They optimise subject to constraints. In GTEM, the conditions for their optimal solutions are put together with other auxiliary accounting conditions and constraints. These equations are written in a linearised form in which the variables are expressed either in percentage change or change terms. The following three basic rules may be employed to derive a linearised equation (that is, in percentage change terms) from its level form: the product rule, the sum rule and the power rule.

The product rule

Using logarithmic differentiation, a product expressed in levels is translated into the sum of the percentage changes in the components of the product. For example, let:

$$V = PX$$

where X is the quantity of a commodity, P its price and V its value, which results in the following expression after taking logarithmic differentials:

$$d \log V = d \log P + d \log X,$$

which can be written as:

$$dV/V = dP/P + dX/X$$

and multiplying through by 100 gives the linearised equation in percentage change form:

$$v = p + x$$

where v, p and x are respectively the percentage changes in V, P and X, the variables expressed in level terms. Similarly, a division of two level variables can be translated into the difference of the percentage changes in the components of the division. That is:

$$X = V/P \Rightarrow x = v - p.$$

The sum rule

An additive (or difference) relation in the levels of the variables results in a share-weighted expression when expressed in percentage changes. For example, differentiating the following simple relation between income, consumption and savings:

$$Y = C + S,$$

where Y is income, C is consumption and S is savings, yields:

$$dY = dC + dS.$$

In terms of relative changes:

$$Yy = Cc + Ss$$

where y, c and s are the percentage changes in the level variables Y, C and S respectively. Most linear sums or differences in GTEM are expressed in this form.

Alternatively, the above equation may also be written as:

$$y = S_c c + S_s s,$$

where $S_c = C/Y$ and $S_s = S/Y$ are the shares of each component in the levels total.

This rule applies to many national accounting equations — sums of aggregates expressed in levels become share-weighted sums of the percentage changes in the components. It also applies to, for example, the zero pure profit and market clearing conditions.

The power rule

Linearisation of a relation in which a variable is raised to a power results in the product of the percentage change in the variable and its power. For example:

$$Y = \beta X^{\alpha} .$$

A logarithmic differentiation of both sides yields:

$$y = \alpha x ,$$

where y and x are percentage changes in Y and X, the variables are expressed in level terms, and a and β are constants.

For example, government demand in GTEM is modelled as resulting from maximising a Cobb-Douglas utility function U subject to the budget constraint Y, given prices. This problem can be written as:

$$\text{Max } U = \beta \prod_{i=1}^{n} X_i^{\alpha_i}; \quad \sum_{i=1}^{n} \alpha_i = 1; \qquad \text{subject to } \sum_{i} P_i X_i = Y$$

The first order conditions for the maximum together imply that the demand for each commodity i is given by:

$$X_i = \alpha_i Y / P_i$$

From this solution, it can easily be seen that α_i is the share of commodity i on the budget and therefore remains constant as long as the utility function does not change. It is easy to see, using the product rule, that the linearised form of this equation is simply:

$$x_i = y - p_i$$

where the lowercase letters indicate the linearised form (percentage change form) of the corresponding uppercase letters representing the level values of the variables.

Note that GTEM is a collection of linearised equations, which are mainly derived from the first order condition of the optimisation problem of various agents, equilibrium conditions and accounting identities.

Modelling technical change in GTEM

GTEM uses a particular way of introducing technical changes that is very similar to the approach developed in Dixon (1982) and Hertel (1997). This section provides a simple example of how the technical changes are represented in the model. First input demand functions are derived for a firm facing a CES production function and then its input demand function is derived when technical changes are present.

Input demand without technical change

First consider an agent's (such as a producer's) cost minimisation problem facing a CES aggregator (production) function without a technical change:

(2.16) $$\min \; Y = P_1 X_1 + P_2 X_2$$

subject to $$X = (\beta_1 X_1^{\alpha} + \beta_2 X_2^{\alpha})^{1/\alpha}$$

The Lagrangian function can be written as:

(2.17) $$L = Y + \lambda\left[X - (\beta_1 X_1^{\alpha} + \beta_2 X_2^{\alpha})^{1/\alpha}\right]$$

where λ is the shadow cost to the objective of an extra unit of aggregate X defined in the constraint.

The first order conditions are:

(2.18) $$\frac{\partial L}{\partial X_i} = P_i - \lambda\beta_i X_i^{\alpha-1} X^{1-\alpha} = 0$$

(2.19) $$\frac{\partial L}{\partial \lambda} = X - (\beta_1 X_1^{\alpha} + \beta_2 X_2^{\alpha})^{1/\alpha} = 0.$$

Multiplying through the right hand part of equation (2.18) by X_i and summing over i gives:

(2.20) $$Y - \lambda X^{1-\alpha}\left[\beta_1 X_1^{\alpha} + \beta_2 X_2^{\alpha}\right] = 0.$$

Since the square bracket term is equal to X^{α} this simplifies to:

(2.21) $$\lambda = Y/X = P$$

where P is defined as the price of the aggregate X, since it is the expenditure on X divided by the quantity commodity aggregate.

Dividing the first order condition (2.18) by P (that is by λ) and rearranging yields:

(2.22) $$\frac{P_i}{P} = \beta_i (X / X_i)^{1-\alpha}.$$

Let $\sigma = (1-\alpha)^{-1}$. Taking logarithms of both sides in equation (2.22) yields:

$$\sigma(\ln P_i - \ln P) = \sigma \ln \beta_i + \ln X - \ln X_i.$$

Holding σ and βi constant, as they are model parameters, and differentiating yields the following expression in percentage changes:

(2.23) $$x_i = x - \sigma(p_i - p)$$

where lower case letters are the percentage change equivalents of the corresponding upper case variables. The Armington part of the intermediate and final demand equations in the model follow this structure.

Input demand with technical change

Assume that the production function is in fact defined over the efficiency units, rather than on physical units, but the producer can only buy or hire physical units of the inputs. In this case the cost minimising problem of the producer can be restated as:

(2.24) $$\min \; Y = P_1 X_1 + P_2 X_2$$

subject to $$X = A_0[\beta_1 (A_1 X_1)^{\alpha} + \beta_2 (A_2 X_2)^{\alpha}]^{1/\alpha},$$

where A_i represents the efficiency units embodied in a physical unit of input i and A_0 is the input neutral efficiency parameter.

The minimisation problem (2.24) can be rewritten as:

(2.25) $$\min \quad \sum_i P_i^* X_i^*$$

subject to $$X^* = [\beta_1 (X_1^*)^\alpha + \beta_2 (X_2^*)^\alpha]^{1/\alpha} .$$

Where:

(2.26) $$X_i^* = A_i X_i$$

(2.27) $P_i^* = P_i / A_i$ and

(2.28) $$X^* = X / A_0 .$$

Note that $$S_i^* \equiv \frac{P_i^* X_i^*}{\sum_k P_k^* X_k^*} = \frac{P_i X_i}{\sum_k P_k X_k} \equiv S_i$$

Clearly, the minimisation program (2.25) is similar to the program (2.16) and hence the solution will be similar to equation (2.23).

So:

(2.29) $$x_i^* = x^* - \sigma(p_i^* - p^*) .$$

By using the linearised form of the definitions (2.26) and (2.27) in equation (2.29), then:

(2.30) $$x_i + a_i = x - a_0 - \sigma(p_i - a_i - p^*)$$

where the percentage change in the unit price can be written as:

(2.31) $$p^* = \sum_k S_k^* p_k^* = \sum_k S_k p_k^* = \sum_k S_k [p_k - a_k] .$$

Thus equation (2.30) yields the demand function for physical units of inputs, where the underlying technology defines inputs in efficiency units. Technical change modelled here alters the relationship between a physical unit and the number of efficiency units it represents. When an input becomes more productive, it embodies a larger quantity of efficiency units. At unchanged prices for the physical units the corresponding prices faced by the firms for their efficiency units fall. GTEM makes frequent uses of the form of equations (2.30) and (2.31) in expressing input demand functions with technical change.

Definition of GTEM sets

Table 2.2 contains lists of sets used in GTEM and their subsets. The elements are, of course, aggregation dependant. The elements given here are examples only but the set definitions may serve a reference.

Table 2.2: **Sets used in GTEM**

A. SETS	**Description**	**Elements (for example)**
REG	Regions (that partition the world)	{aus, nzl ,usa, can, jap, … ,fsu, cea, row}
NSAV_COMM	Non-savings commodities (universal set of commodities)	{land, labour, capital, nat res, col, oil,…,manu, svce, capital goods}
TRAD_COMM	Tradable commodities - (non-savings commodities excluding primary factors and capital goods)	{col, oil, … ,manu, svce}
ENDW_COMM	Endowment commodities (primary factors only)	{land, labour, capital, nat res}
DEMD_COMM	Demanded commodities (endw_comm + trad_comm)	{land, labour, capital, nat res, col, oil,…, manu, svce}
PROD_COMM	Produced commodities (demd_comm – endw_comm)	{col, oil, … ,manu, svce, capital goods}
ENDWS_COMM	Sluggish endowment commodities	{land}
ENDWNCOMM	Natural resources	{nat res}
ENDWM_COMM	Mobile endowment commodities	{labour, capital}
CGDS_COMM	Capital goods commodities (for investment)	{capital goods}
ENDWC_COMM	Endowment commodity – capital	{capital}
ENDWL_COMM	Endowment commodity – labour	{labour}
ENDWLn_COMM	Endowment commodity – land	{land}
TECH_IND	Industries with technology bundles	{egw, i_s}
EGW	Electricity sector	{ely}
I_S	Iron and steel sector	(i_s}
TECH	Technologies	{t1, t2, t3, t4, t5, t6,…}
CONVTECH_ELY	11 conventional power generating technologies	{t1_12, t2_13, t3, t4_14, t5_t11}
BCOLTECH_ELY	2 brown coal fired power generating technologies	{t1, t12}
SCOLTECH_ELY	2 steaming coal fired power generating technologies	{t2, t13}
GASTECH_ELY	2 gas fired power generating technologies	{t4, t14}
MULTITECH_ELY	Technologies with two branches: with and without CCS	{t1_11, t2-1, t4_t14}
CLANER_TECH	Cleaner power generation technologies – excluding nuclear and hydro but including fossil fuel with CCS	{t7_t14}
OTHER_TECH	Other power generation technologies	= TECH – CLEANER_TECH

CAPTURE_TECH	Technologies with CCS	{t12_t14}
CONVEN_TECH	Conventional technologies	= TECH – CAPTURE_TECH.
RENEW_TECH	Renewable technologies, excluding hydro	{t7_t11}
NRENEW_TECH	Nonrenewable technologies (including hydroelectric technology)	= TECH – RENEW_TECH
TECH_COMM	Inputs used by tech_ind	{nfm, omn, bcol, scol, gas, p_c, egw, labour, capital}
TECH_COMM_T	Traded commodities used by technologies	{nfm, omn, col, oil, gas, p_c, egw}
TECH_COMM_F	Factors used by technologies	{labour, capital}
GHG	Greenhouse gases	{CO2, CH4, etc}
FUEL	Fuel commodities	{bcol, ckcol, scol, oil, gas, P_C, Ely}
NOT_FUEL	Traded commodities other than fuel commodities	TRAD_COMM – FUEL.
FUEL_COMM	A single fuel composite commodity	{ener}
CON_COMM	Consumer goods at top level	FUEL_COMM+ NOT_FUEL
GAS	Gas	{gas}
FFUEL_COMM	Primary fossil fuel sectors	{bcol, ckcol, scol, oil, gas}
EMIS_COMM	Commodities producing (combustion) emissions when used	{bcol, ckcol, scol, oil ,gas, p_c, crp}
EMIS_COMM_T	TECH_COMM producing (combustion) emissions when used	{bcol, ckcol, scol, oil, gas, p_c}
EMIS_IND	Industries producing noncombustion emissions	{bcol, ckcol, scol, oil, gas, p_c, nfm, crp, nmm, agric}
CLEAN_IND	Industries that do not produce noncombustion emissions	= PROD_COMM- EMIS_IND
RES_IND	Natural resource based indusrties	{bcol, ckcol, scol, oil, gas, omn, fishnchi}
NON_RES_IND	Industries not based on natural resources	= PROD_COMM – RES_IND
EX_RES_IND	Resources based on exhaustible natural resource	{bcol, ckcol, scol, oil, gas, omn}
OTHER_IND	Industries not based on exhaustible natural resources	= PROD_COMM – EX_RES_IND
AGG_IND	Agricultural sectors (ag land users)	{AGRIC}
NON_AGG_IND	Nonagricultural sectors	= PROD_COMM – AGG_IND
NF_TRAD_COMM	Clean traded commodities: that do not emit emissions	= TRAD_COMM - EMIS_COMM
NTECH_COMM_T	Traded commodities not in TECH_COMM	= TRADED_COMM - TECH_COMM_T
NONF_TECHCOM	Non-fossil tech commodities	= TECH_COMM_T – EMIS_COMM_T
TECHM_COMM_F	Mobile factors in tech_comm	= ENDWM_COMM
NON_NATRES	Demanded commodities excluding natural resources	= DEMD_COMM – ENDWN_COMM

Table 2.2: **Sets used in GTEM** *(continued)*

SUBSETS	SET
PROD_COMM	$\subset$ *NSAV_COMM*
DEMD_COMM	$\subset$ *NSAV_COMM*
ENDW_COMM	$\subset$ *DEMD_COMM*
TRAD_COMM	$\subset$ *DEMD_COMM*
TRAD_COMM	$\subset$ *PROD_COMM*
CGDS_COMM	$\subset$ *PROD_COMM*
ENDWS_COMM	$\subset$ *ENDW_COMM*
ENDWN_COMM	$\subset$ *ENDW_COMM*
ENDWM_COMM	$\subset$ *ENDW_COMM*
ENDWC_COMM	$\subset$ *NSAV_COMM*
ENDWL_COMM	$\subset$ *ENDW_COMM*
ENDWLn_COMM	$\subset$ *ENDW_COMM*
TECH_COMM	$\subset$ *DEMD_COMM*
TECH_IND	$\subset$ *TRAD_COMM*
EGW	$\subset$ *TECH_IND*
GAS	$\subset$ *TRAD_COMM*
FFUEL	$\subset$ *TECH*
FUEL	$\subset$ *TECH_COMM*
FFUEL_COMM	$\subset$ *TRAD_COMM*
EMIS_COMM	$\subset$ *TRAD_COMM*
EMIS_COMM_T	$\subset$ *EMIS_COMM*
EMIS_COMM_T	$\subset$ *TECH_COMM_T*
EMIS_IND	$\subset$ *PROD_COMM*

Table 2.2: **Sets used in GTEM** *(continued)*

SETS used in the DEMOGRAPHIC MODULE of GTEM		
REG_VARS	Variables used in regressing life expectancy	{income, dummy1, dummy2, dummy3}
INC_GROUP	**Income group**	**{Low, medium, high}**
GENDER	Gender	{male, female}
AGE	All age groups (intertemporal)	{0 –1, 1-2, …, 99–100}
*AGE*0	New born	{0–1}
*AGE*1	All age groups except 0–1	{1–2, 2–3, …, 99–100}
*AGE*2	Last age group	100+
EXAMPLE_REG	A reference region for post transition fertility behaviour	{usa}
WORK_AGE	Working age	{16–17, 17–18, …64–65}
INC_GROUP	Income groups of regions	{low, medium, high}
SUBSETS		
*AGE*1	$\subset$ AGE	
*AGE*2	$\subset$ AGE1	
*AGE*0	$\subset$ AGE	
WORK_AGE	$\subset$ AGE1	
SETS used in the ENVIRONMENT MODULE of GTEM		
EMISSIONS	All types of emissions	
GHG	Greenhouse gases	
LGWP_GHG	Low global warming potential GHGs	{CO_2, CH_4, N_2O}
HGWP_GHG	High global warming potential GHGs	
CO2	Carbon dioxide	{co2}
SCHEMES	Carbon trading blocks	{b1,b2}
CRP_COMM	Commodity releasing N_2O when used	{crp}
POP_ECON	Economies where HGP_GHGs move with population	
PCGDP_ECON	Regions where HGWP_GHGs move with PCGDP	
AEOPAR	Set of parameters of emissions response function	{a,b,c,d,e}
HGWP_PAR	Set of parameters of the response functions of HGWP_GHGs	{a,d}
SUBSETS		
GHG	$\subseteq$ EMISSIONS	
NON_GHG	=EMISSIONS - GHG	
LGWP_GHG	$\subset$ GHG	
HGWP_GHG	= GHG – LGWP_GHG	
HGWP_PAR	$\subset$ AEOPAR	