Analytical distributions and statistics for managing import pathway risk using the continuous sampling plans CSP-2 and CSP-3

B. Barnes

Working Paper 22.04

Research by the Australian Bureau of Agricultural and Resource Economics and Sciences

August 2022

Cataloguing data

This publication (and any material sourced from it) should be attributed as: Barnes, B, 2022, *Analytical distributions and statistics for managing import pathway risk using the continuous sampling plans CSP-2 and CSP-3*, ABARES Working Paper, Canberra, August, DOI: https://doi.org/10.25814/5gea-1h90 CC BY 4.0.

ISSN 189-3128

Australian Bureau of Agricultural and Resource Economics and Sciences (ABARES)
GPO Box 858 Canberra ACT 2601
Telephone 1800 900 090
Web agriculture.gov.au

Acknowledgement of Country

We acknowledge the Traditional Custodians of Australia and their continuing connection to land and sea, waters, environment and community. We pay our respects to the Traditional Custodians of the lands we live and work on, their culture, and their Elders past and present.

Analytical distributions and statistics for managing import pathway risk using the continuous sampling plans CSP-2 and CSP-3

Belinda Barnes

Australian Bureau of Agricultural Resource Economics and Sciences, ABARES
Canberra, Australia
Correspondence: Belinda.Barnes@agriculture.gov.au

August 1, 2022

Executive summary

This is a technical paper intended to supplement an earlier technical paper [1] that considered the specific continuous sampling plan CSP-1. Two alternative plans are analysed here — CSP-2 and CSP-3, which are designed to reduce intervention on low-risk pathways. Our motivation is to generalise the work on CSP-1 to provide efficient and practical tools that can effectively underpin and guide robust design-decisions across a range of continuous sampling plans.

Continuous sampling plans (CSP) are operational biosecurity systems commonly implemented to reduce the risk of exotic pest and disease introductions across the border. Such systems are designed to reward importers who regularly supply compliant goods by reducing the frequency of border inspections, thereby lowering costs for importers and facilitating the rapid transit of cargo. A CSP-1 system is currently in place for Australian prawns returning from an approved offshore facility (Pers. Comm. Animal Biosecurity DAFF), while CSP-3 is applied at the border to screen consignments of plant-based material, such as coffee beans and dried apricots [14]. CSP-2 is not currently implemented but Compliance-Based Intervention Schemes (CBIS) are under review. Both CSP-2 and CSP-3 are specifically designed to provide importers with a 'second chance'. These plans recognise that a single, rare detection for characteristically compliant importers can lead to major delays in the transit of goods — an increase in cost that offers little benefit for biosecurity.

The purpose of this work, together with that in [1], is to improve the reliability of CSP design for a range of operational systems. In contrast to earlier approaches based on mean values, simulations or the analysis of Markov chains, we formulate simple algebraic expressions for the distributions and statistics associated with all processes in a complete cycle of CSP-1, CSP-2 and CSP-3. This includes expressions for variance (uncertainty), for the probability that leakage occurs, and distributions for the volume of leakage. Results are fast, accurate and straightforward to calculate, and they properly capture the effect of uncertainty and rare events, which are important features in low-prevalence settings and thus highly relevant to pathway analysis. Analytical solutions also explicitly express how inspection-sensitivity, system drivers and available controls interact and affect results. These insights have the potential to influence biosecurity outcomes and improve the reliability of operational systems.

Our formulations provide foundational 'building-blocks' for the design and analysis of a range

of CSP systems that include both uncertainty and inspection sensitivity. Collectively, results contribute an efficient and easily accessible approach that has not previously been available:

- a fast and accurate means of analysis, particularly when prevalence is low — which is commonly the case for imported and exported consignments of goods
- expressions for variance, so that it becomes straightforward to understand the influence of system uncertainty on results, and to integrate it into CSP design principles. Current methods often rely on mean values with variance not easily available
- equations that express precisely how CSP system controls (census numbers, monitoring fractions and inspection-effort, which may be different for each mode) affect leakage and uncertainty, which has important consequences for efficient and robust system design. These inter-relationships can be difficult to determine using simulation methods
- results that take inspection sensitivity (imperfect detection) into account are provided for all distributions and statistics. This is particularly important when designing Australian biosecurity systems because consignments arriving at the border are sampled and inspected to detect contamination. This is not a perfect detection process, which has direct consequences for system design [1].

The above information is critical for decision-making under uncertainty, but can be difficult, time-consuming and/or computationally expensive to achieve using the more routinely applied methods.

Our analytical results are intended to replace or complement alternative methods of analysis. In general they replace the need for simulations and long-run Markov chain approaches. Alternatively, they can be embedded within more complex simulations and Markov chain decision processes. Analytical results, such as those provided here, can improve the accuracy and efficiency of these analyses, and considerably increase the information available for management decisions.

The distributions and statistics provided in this paper have been specifically formulated to support Australian biosecurity systems. We highlight, however, that our approach has more general relevance, with results not restricted to the example schemes of CSP-1, CSP-2 and CSP-3. It is straightforward to apply methods and results to assess alternative arrangements of few or many of the different CSP modes — and thus to evaluate a far broader range of alternative plans.

There are also numerous extensions to the foundational results formulated in this paper — which are the subject of future work. Economic aspects are not directly considered, but models are constructed so that the integration of costs and benefits is straightforward, and uncertainty concerning prevalence is easily included in the stochastic model. An extensive analysis of tradeoffs between system controls and alternative CSP measures and strategies is considered beyond the scope of this work. Different biosecurity divisions will have distinct practical constraints and priorities, which need to be understood and taken into account. This work, however, provides practical and appropriate tools for that purpose.

Our results, together with those in [1], contribute an efficient means of assessing and designing robust CSP systems for biosecurity. The statistical approach proposed is more powerful than standard simulation methods for many current applications — while, at the same time, remaining fast, simple, accurate and accessible.

Contents

Abstract

Australia imposes regulations on goods that arrive in the country and have the potential to introduce exotic pests and diseases. Continuous sampling plans (CSP) are operational biosecurity systems commonly implemented at the border to manage risk and, simultaneously, keep regulatory inspection costs low. Currently, CSP-1 systems are underpinned by the results and proposed design-criteria in the classical work of Dodge (1943) [4] and CSP-2 and CSP-3 systems are underpinned by that in Dodge and Torrey (1951) [5]. In [1] we extended that foundational work for CSP-1 to include full distributions, uncertainty and inspection sensitivity, and in this paper, which supplements that work, we extend the foundational work for CSP-2 and CSP-3 in the same way. The inclusion of inspection sensitivity and uncertainty are both highly relevant to Australian border operations where samples from arriving consignments, with low but variable levels of contamination, are selected and inspected. We provide analytical distributions and statistics for all processes, for each mode and for the full cycle of each CSP system, including expressions for variance and leakage. Results, together with those from [1], provide a highly efficient and easily accessible means of designing a range of CSP schemes, increasing the information available for decisions, and having the potential to influence biosecurity outcomes and improve the reliability of operational systems.

1 Introduction

Australia imposes regulations on goods that arrive in the country and have the potential to introduce exotic pests and diseases. For some products, these regulations require inspections or treatments which may be costly or delay delivery resulting in further costs to importers. To minimise costs while managing biosecurity risks, the Australian Department of Agriculture, Fisheries and Forestry runs a Compliance-Based Intervention Scheme which can reduce inspection rates for importers of specified products who demonstrate consistent compliance with Australia's biosecurity requirements. Compliant importers benefit from the scheme through smoother clearance of goods at the border and reduced regulatory costs.

The Compliance-Based Intervention Scheme (CBIS) is implemented at the border, using continuous sampling plans (CSP) to monitor levels of contamination. When applied to a pathway (such as goods of a specified type from a particular country), CSP requires separate inspection histories for each sub-pathway and reducing the proportion of consignments inspected where consistent compliance is demonstrated — where a sub-pathway here may be any identifiable partition, such as an importer, supplier, or a defined place where goods are produced.

The simplest continuous sampling scheme is the two stage plan, CSP-1. This scheme was originally proposed by Dodge in 1943 [4] and it underpins a number of more recently proposed alternatives — such as, the more complex variants CSP-2 and CSP-3 [5]. All three schemes assume a perfect detection process. CSP-1 and CSP-3 are currently implemented in Australian biosecurity systems, with CSP-1 used to test consignments of Australian prawns returning from offshore facilities, and CSP-3 applied at the border on low-risk pathways to screen consignments of plant-based material, such as coffee beans and dried apricots [14]. Both the CSP-2 and CSP-3 schemes are specifically designed to provide importers with a 'second chance', or less severe consequences for occasional non-compliance. Relative to CSP-1, these plans recognise that a single, rare detection for a characteristically compliant importer can lead to major delays in the transit of goods, and increased costs — interventions and costs that offer little benefit for biosecurity.

The CSP-1 system has two inspection modes — a 'census mode' or 'enhanced inspection mode', in which 100% of arriving items (consignments of goods) are inspected until a specified number (the clearance number) of sequential consignments pass the inspection process. Importers then transition to 'monitoring mode' in which a lower proportion (the monitoring fraction) of consignments is inspected. Importers remain in monitoring mode unless contamination is detected during an inspection, whereupon they are returned to census mode.

The CSP-2 scheme is a 3-mode system that includes an additional sampling mode in CSP-1 —

to which importers transition when contamination is detected in monitoring mode. Compliant importers can then return to monitoring mode without having to return to census mode. The CSP-3 scheme is a 4-mode system that includes a further mode into CSP-2 in which all (100%) of the next 4 (or other fixed number) arrivals are inspected — to which importers transition when contamination is detected in monitoring mode. In this case compliant importers transition to the second sampling mode, and then to monitoring mode without having to return to census mode.

Dodge (1943) and Dodge and Torrey (1951) (see also [7, 11]) derived analytical expressions for certain expected values associated with CSP-1, CSP-2 and CSP-3. They assume a perfect detection process, meaning that an inspection detects contamination when present with certainty, and the CSP design principles they propose are underpinned by these expected values and assumptions. The process they model is the inspection of individual units in a production line, and they do not take model uncertainty or inspection sensitivity into account. Results from their analysis currently guide the design of biosecurity systems implemented at the Australian border (see, for example, [3]).

This paper is intended to supplement [1]. That paper extends and generalises the foundational work of [4] for CSP-1, and in this paper we generalise results for the more complex CSP-2 and CSP-3 variants. The primary purpose is to guide robust design principles for use across the range of continuous sampling plans applied in Australian biosecurity operations. Our analysis explicitly incorporates the effect of inspection sensitivity because, for consignments of goods arriving at the border, sampling schemes and inspection processes are seldom perfect. We extend the work of [4, 5] using standard statistical methods — generating functions and conditioning — to formulate accessible, algebraic expressions for full distributions and statistics associated with the processes and cycles of CSP-1, CSP-2 and CSP-3. This includes expressions for variance (uncertainty). For many applications, our approach replaces the need for simulations or long-run Markov chain methods (as in, for example, [9, 12]), and are likely to simplify Markov decision processes [15]. Results are fast, accurate and straightforward to calculate, and they properly capture the effect of uncertainty and rare events, which are important features in low-prevalence settings and thus highly relevant to pathway analysis. Analytical solutions also explicitly express how system drivers and available controls interact and affect results. These insights can be difficult to achieve using simulations, but have the potential to influence biosecurity outcomes and improve the reliability of operational systems. An in-depth analysis of new and optimal CSP design strategies, however, is considered beyond the scope of this work.

The paper is organised as follows. Foundational stochastic models for each mode of the CSP-2 and CSP-3 systems are constructed in Section 2, and then combined in Section 3 to formulate stochastic models and statistics for a full cycle of each system. Analytical expressions for all expected-value results are summarised in Section 4. We then discuss future work and provide concluding remarks in Section 5.

2 Stochastic models for each mode of CSP-2 and CSP-3

The CSP-3 biosecurity system has four distinct modes. A schematic diagram of the system is provided in Figure 1, where allowable transitions between the different modes are identified using arrows. The four modes are characterised as follows:

1. **Mode 1 (Census Mode or Enhanced-Inspection Mode)** — 100% of all arriving consignments are inspected. When no contamination is detected in a fixed number (the clearance number) of sequential inspections, the importer transitions to Mode 2 (Monitoring Mode).
2. **Mode 2 (Monitoring Mode)** — a fixed proportion (the monitoring fraction) of arriving consignments is inspected. If contamination is detected, the importer transitions to Mode 3.

3. **Mode 3** — 100% of consignments are inspected. When no contamination is detected in a fixed number (second clearance number, or tight census number) of inspections, the importer transitions to Mode 4. If contamination is detected during this fixed number of inspections, the importer is retuned to Mode 1 (Census Mode) and the full process begins again.
4. **Mode 4** — a fixed proportion (second monitoring fraction) of consignments are inspected. When no contamination is detected in a fixed number (third clearance number) of inspections, the importer is returned to Mode 2 (Monitoring Mode). If contamination is detected during this fixed number of inspections, the importer is returned to Mode 1 (Census Mode) and the full process begins again.

We assume, as is common practice [9, 14], that importers start in Census Mode (Mode 1). Compliant importers typically transition to Monitoring Mode (Mode 2) and remain there. Alternatively, relative to the CSP-1 system, CSP-3 provides a 'second chance' to 'mostly-compliant' importers, and they may transition to Monitoring Mode and then between Modes 2, 3 and 4 without ever returning to Census Mode (Mode 1). Non-compliant importers will be returned to Census Mode (Mode 1) via Modes 3 and 4.

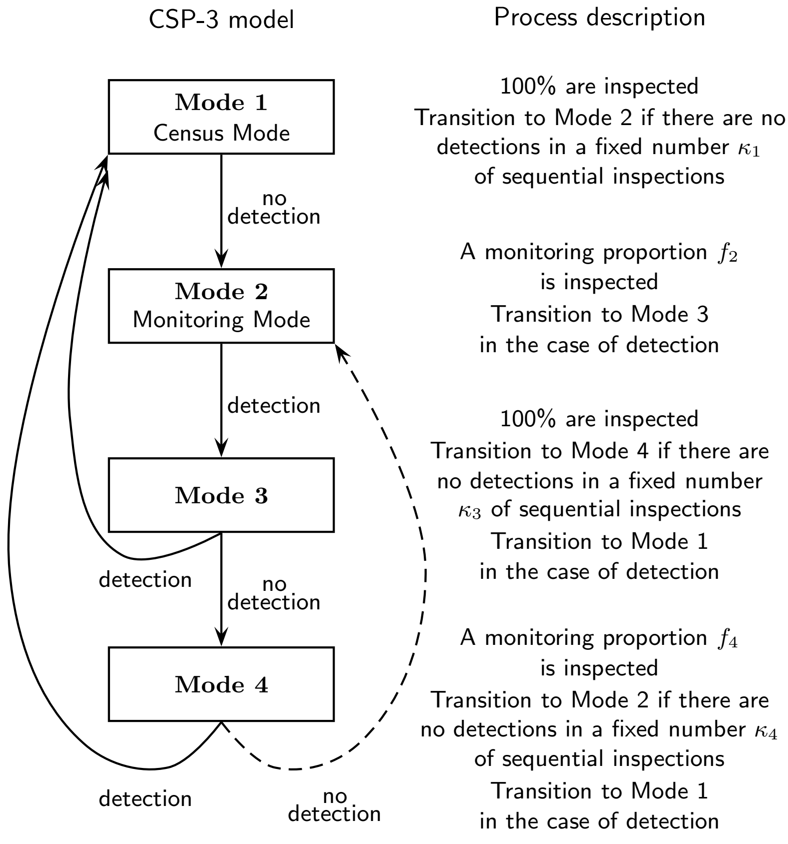

Figure 1: Schematic diagram of the CSP-3 system originally proposed in [5]. The inspection process begins in Mode 1 (Census Mode) and a full cycle is complete with a return to Mode 1 (Census Mode) — outer solid curves on the left-hand-side. The CSP-2 system excludes Mode 3, but is otherwise identical (see Appendix Figure D.3).

The CSP-2 biosecurity system has three distinct modes — Mode 1 (Census Mode) and the two sampling modes (Modes 2 and 4). Thus it is identical to CSP-3 with the *exclusion* of Mode 3. The three modes are characterised as above and a schematic diagram of the system is provided in Appendix Figure D.3.

Symbol	Description
$\mathbb{E}$	expected value
$\mathbb{V}\text{ar}$	variance
$\mathbb{P}$	probability
$\Phi_X(s)$	probability generating function (pgf) for generic random variable (rv) X
p	probability that an arriving consignment is contaminated
κ_j	number of sequential, inspected consignments required to transition ($j = 1, 3, 4$, with κ_1 the clearance number in Census Mode)
f_j	proportion of arriving consignments inspected ($j = 2, 4$)
$\mathbb{D}_{\kappa_j}$	event that contamination is detected in at least one of κ_j inspections
$\overline{\mathbb{D}}_{\kappa_j}$	event that contamination is not detected in κ_j inspections
$A_j, A_{j\delta}$	rv for the number of consignment arrivals in a pass through Mode j ($j = 1, 2, 3, 4$)
$I_j, I_{j\delta}$	rv for the number of inspections in a pass through Mode j ($j = 1, 2, 3, 4$)
$L_j, L_{j\delta}$	rv for the leakage in a pass through Mode j ($j = 1, 2, 3, 4$)
$H\vert\mathbb{D}_{\kappa_j}$	rv for the consignment-number of first-detection in a sequence of κ_j inspections, conditional on detection in the sequence
δ	probability that contamination is detected when present ($0 < \delta \leq 1$)

Table 1: Notation, random variables and parameters associated with the CSP-3 biosecurity system (Figure 1), and the CSP2 system (Figure D.3), where subscripts 1,2,3 and 4 identify the mode associated with parameters and random variables, subscript δ identifies expressions that take inspection sensitivity into account, and subscript δ_i ($i = 1, 2$) identifies expressions for which inspection sensitivity is taken into account and contributes to a specified component (i) of leakage. Abbreviations: pgf — probability generating function; rv — random variable.

We first construct distributions, expected values and variances for a single pass through each of the distinct modes associated with each system. A pass through a given mode is defined here to begin when an importer transitions to that mode, and ends when they transition from that mode. Then, assuming all importers start in Mode 1 (Census Mode) and that a full cycle includes all transitions and inspection processes until the importer is returned to Mode 1 (Census Mode), we also construct the distribution, expected value and variance for a complete cycle through the CSP-2 and CSP-3 systems with all modes combined, where there may be multiple passes through some modes (Mode 2, Mode 3 (in the case of CSP-3), and Mode 4).

2.1 Distributions and statistics for each mode — with perfect detection

Initially we assume a perfect detection process, meaning that contamination when present is detected during an inspection with certainty. As discussed above, this assumption underpins the foundational work of [4, 5], and also the simulation studies undertaken by ACERA/CEBRA which apply directly to Australian biosecurity systems (see, for example, [14]). Note, however, that it is straightforward to include an imperfect detection process (inspection sensitivity), as in [1], which is accomplished in Section 2.2 below. This inclusion is highly relevant to current sampling strategies implemented at the Australian border, for which detection may be imperfect.

As in [1], results are derived using probability generating functions. They offer a simple, elegant and particularly powerful means of analysis, enabling the construction of mixed and compound distributions that combine classes of stochastic processes, and from which it is straightforward to determine the moments. Some basic principles are provided in [1] (Appendix A), but a more detailed discussion can be found in most standard statistical texts — for example, [6]. We also note that our results are intended to contribute to system-design criteria 'over the long-run'. For small, specific numbers of arriving consignments, state transition matrices (Markov chains) offer a related method (see, for example, [12, 13]). Our approach, however, also formulates statistics that are relevant to this particular application.

Let p denote the probability that a randomly selected consignment arriving at the border is contaminated. Let κ_j, with $j = 1, 3$ and 4 for each of Modes 1, 3 and 4, respectively, denote a restricted sequence length. In Mode 1 (Census Mode), κ_1 is the standard clearance number (denoted i in [1, 4, 5]); in Mode 3, κ_3 is the number of successive inspections without detection required to transition to Mode 4; and in Mode 4, κ_4 is the number of successive inspections without detection required to transition to Mode 2 (see Figure 1). Let f_j be the proportion of arriving consignments inspected. Trivially, for Modes 1 and 3, $f_1 = f_3 = 1$ because all arriving consignments are inspected (f_1 and f_3 are not explicit in our formulations). For Mode 2 (Monitoring Mode) and Mode 4, f_2 and f_4 are monitoring fractions, respectively — which may take on different values and are explicit in our formulations.

Define $\overline{\mathbb{D}}_\kappa$ as the event that there are no detections in a sequence of κ inspections, and define $\mathbb{D}_\kappa$ as the complementary event. It then follows that, conditional on p,

$$\mathbb{P}(\overline{\mathbb{D}}_\kappa) = (1-p)^\kappa \qquad \mathbb{P}(\mathbb{D}_\kappa) = 1 - (1-p)^\kappa. \tag{1}$$

For each mode j, for $j = 1, 2, 3$ and 4, we define random variables A_j, I_j and L_j for the number of consignment arrivals, the number of consignment inspections, and the number of consignments that leak through the system, respectively, during a single pass through mode j.

We now construct distributions and statistics for arrivals, inspections and leakage in each mode of Figure 1, with conditioning on p understood and noting that results for Modes 1, 2 and 4 are relevant to CSP-2 (Figure D.3). A summary of notation, random variables and parameters used below is provided in Table 1.

2.1.1 Mode 1 — Census Mode

In Mode 1, or Census Mode, all sequentially arriving consignments are inspected until there are no detections in a fixed number, the clearance number κ_1, of sequential inspections (Figure 1).

Full distributions and statistics for the number of arrivals in Mode 1 (random variable A_1), for the number of inspections undertaken in Mode 1 (random variable I_1), and for the leakage through Mode 1 (random variable L_1), are formulated in [1]. In summary, the number of arrivals, which is also the number of inspections, in a single pass through Mode 1 has probability generating function

$$\Phi_{A_1}(s) = \frac{((1-p)s)^{\kappa_1}\left(1-(1-p)s\right)}{1 - s + ps\left(1 - ((1-p)s)^{\kappa_1}\right)} \tag{2}$$

which fully defines the distribution, and from which it is straightforward to establish the expected value and variance

$$\mathbb{E}(\text{arrivals in Mode 1}) = \mathbb{E}(A_1) = \frac{1-(1-p)^{\kappa_1}}{p(1-p)^{\kappa_1}} \tag{3}$$

$$\mathbb{V}\text{ar}(A_1) = \frac{(1-p)^{-2\kappa_1} - p(2\kappa_1+1)(1-p)^{-\kappa_1} - (1-p)}{p^2}. \tag{4}$$

When detection is assumed perfect, as in this analysis, there is no leakage through Mode 1.

The above expected value agrees with that provided in [4, 5].

2.1.2 Mode 2 — Monitoring Mode

In Mode 2, Monitoring Mode, a fixed proportion, the monitoring fraction f_2, of all sequentially arriving consignments is inspected until a contaminated consignment is detected (Figure 1).

Full distributions and statistics for the number of arrivals in a single pass through Mode 2 (random variable A_2), for the associated number of inspections undertaken in Mode 2 (random variable

I_2), and for the leakage through Mode 2 (random variable L_2), are formulated in [1]. In summary, the probability generating functions are given by

$$\Phi_{A_2}(s) = \frac{pf_2 s}{1-(1-pf_2)s} \qquad \Phi_{I_2}(s) = \frac{ps}{1-(1-p)s} \qquad \Phi_{L_2}(s) = \frac{f_2}{1-(1-f_2)s} \tag{5}$$

with expected values and variances

$$\mathbb{E}(\text{arrivals in Mode 2}) = \mathbb{E}(A_2) = \frac{1}{pf_2} \qquad \mathbb{V}\text{ar}(A_2) = \frac{1-pf_2}{(pf_2)^2} \tag{6}$$

$$\mathbb{E}(\text{inspections in Mode 2}) = \mathbb{E}(I_2) = \frac{1}{p} \qquad \mathbb{V}\text{ar}(I_2) = \frac{1-p}{p^2} \tag{7}$$

$$\mathbb{E}(\text{leakage through Mode 2}) = \mathbb{E}(L_2) = \frac{1-f_2}{f_2} \qquad \mathbb{V}\text{ar}(L_2) = \frac{1-f_2}{f_2^2}, \tag{8}$$

noting that the number of inspections in a single pass through Mode 2 is independent of the monitoring fraction (f_2), and that leakage is independent of prevalence (p). Leakage, when detection is assumed perfect, can occur through Mode 2 because not all consignments are inspected.

The above expected values agree with those provided in [4, 5].

2.1.3 Mode 3

In Mode 3, all arriving consignments are inspected in sequence and the total number of inspections is restricted to κ_3 (Figure 1). This mode was not considered in [1], so full details are provided below.

The probability of no detections in κ_3 sequential inspections is (see Equation (1))

$$\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3}) = (1-p)^{\kappa_3}.$$

Conditional on this event, the number of arrivals, also the number of inspections, is κ_3.

Alternatively, the probability of detection during this sequence of κ_3 arrivals is

$$\mathbb{P}(\mathbb{D}_{\kappa_3}) = 1-(1-p)^{\kappa_3}.$$

Let A_3 denote a random variable for the number of arrivals in Mode 3, which is also the number of inspections (I_3). The conditional number of arrivals has distribution defined by probability generating function (Equation (A.1))

$$\Phi_{A_3|\mathbb{D}_{\kappa_3}}(s) = \Phi_{H|\mathbb{D}_{\kappa_3}}(s) = \frac{ps\left(1-((1-p)s)^{\kappa_3}\right)}{(1-(1-p)^{\kappa_3})\,(1-(1-p)s)}$$

with expected value (Equation (A.2))

$$\mathbb{E}(A_3|\mathbb{D}_{\kappa_3}) = \frac{1-(1-p)^{\kappa_3}\,(1+\kappa_3 p)}{p\left(1-(1-p)^{\kappa_3}\right)} = \frac{1}{p} - \frac{\kappa_3(1-p)^{\kappa_3}}{1-(1-p)^{\kappa_3}}. \tag{9}$$

This expected value agrees with that provided in [5] (h_1 and h_2, Eqs. 4 and 4′).

The total number of arrivals (also inspections) in Mode 3 thus has probability generating function

$$\begin{aligned}\Phi_{A_3}(s) &= s^{\kappa_3}\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3}) + \Phi_{H|\mathbb{D}_{\kappa_3}}(s)\mathbb{P}(\mathbb{D}_{\kappa_3}) \\ &= \Big((1-p)s\Big)^{\kappa_3} + \frac{ps\left(1-\big((1-p)s\big)^{\kappa_3}\right)}{1-(1-p)s}\end{aligned} \tag{10}$$

with expected value and variance

$$\mathbb{E}(A_3) = \frac{1-(1-p)^{\kappa_3}}{p} \tag{11}$$

$$\mathbb{V}\mathrm{ar}(A_3) = \frac{(1-p)\left(1-(1-p)^{2\kappa_3-1} - 2p\kappa_3(1-p)^{\kappa_3} + p(1-2p\kappa_3)(1-p)^{\kappa_3-1}\right)}{p^2} \tag{12}$$

As noted above, the number of inspections is equivalent to the number of arrivals. When detection is perfect, there is no leakage through Mode 3.

2.1.4 Mode 4

In Mode 4, a randomly selected fraction f_4 of sequentially arriving consignments are inspected, and the total number of inspections is restricted to κ_4 (Figure 1). As for Mode 3, this mode was not considered in [1], so full details are provided below.

Consider first the case in which no consignments are detected in κ_4 sequential inspections — which occurs with probability (see Equation (1))

$$\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) = (1-p)^{\kappa_4}.$$

Let A_4 and I_4 denote random variables for the total number of consignment arrivals and inspections in Mode 4, respectively. Consignments are selected for inspection at random, with probability of selection f_4. Conditional on no detections in κ_4 inspections, the number of inspections is fixed — $I_4|\overline{\mathbb{D}}_{\kappa_4} = \kappa_4$ with probability 1. The associated number of arrivals up to and including the κ_4^{th} consignment that is inspected has a negative binomial distribution with probability mass and generating function

$$\mathbb{P}(A_4|\overline{\mathbb{D}}_{\kappa_4}) = \binom{A_4-1}{\kappa_4-1} f_4^{\kappa_4}(1-f_4)^{A_4-\kappa_4} \qquad \Phi_{A_4|\overline{\mathbb{D}}_{\kappa_4}}(s) = \left(\frac{f_4 s}{1-(1-f_4)s}\right)^{\kappa_4} \tag{13}$$

and associated expected value and variance

$$\mathbb{E}(A_4|\overline{\mathbb{D}}_{\kappa_4}) = \frac{\kappa_4}{f_4} \qquad \mathbb{V}\mathrm{ar}(A_4|\overline{\mathbb{D}}_{\kappa_4}) = \frac{(1-f_4)\kappa_4}{f_4^2}. \tag{14}$$

Let $\overline{I}_4$ denote a random variable for the number of arriving consignments in Mode 4 that are *not* inspected. Conditional on the number inspected, $I_4|\overline{\mathbb{D}}_{\kappa_4} = \kappa_4$, the number not inspected has a negative binomial distribution with probability mass and generating function

$$\mathbb{P}(\overline{I}_4|\overline{\mathbb{D}}_{\kappa_4}) = \binom{\overline{I}_4+\kappa_4-1}{\overline{I}_4} f_4^{\kappa_4}(1-f_4)^{\overline{I}_4} \qquad \Phi_{\overline{I}_4|\overline{\mathbb{D}}_{\kappa_4}}(s) = \left(\frac{f_4}{1-(1-f_4)s}\right)^{\kappa_4} \tag{15}$$

and associated expected value and variance

$$\mathbb{E}(\overline{I}_4|\overline{\mathbb{D}}_{\kappa_4}) = \frac{(1-f_4)\kappa_4}{f_4} \qquad \mathbb{V}\mathrm{ar}(\overline{I}_4|\overline{\mathbb{D}}_{\kappa_4}) = \frac{(1-f_4)\kappa_4}{f_4^2}. \tag{16}$$

Note that, conditional on $\overline{\mathbb{D}}_{\kappa_4}$, the expected number of arrivals equals the sum of the expected number of inspections and the expected number not inspected — as would be expected.

Let L_4 denote a random variable for the number of consignment arrivals that leak through the system. Since leakage can only occur when a consignment is not inspected (case of perfect detection), and since each arrival has an equal chance of being contaminated (probability p), the conditional distribution for leakage has generating function

$$\Phi_{L_4|\overline{\mathbb{D}}_{\kappa_4}}(s) = \Phi_{\overline{I}_4|\overline{\mathbb{D}}_{\kappa_4}}(1-p+ps) = \left(\frac{f_4}{1-(1-f_4)(1-p+ps)}\right)^{\kappa_4} \tag{17}$$

with associated expected value and variance

$$\mathbb{E}(L_4|\overline{\mathbb{D}}_{\kappa_4}) = \frac{p(1-f_4)\kappa_4}{f_4} \qquad\qquad \mathbb{V}\mathrm{ar}(L_4|\overline{\mathbb{D}}_{\kappa_4}) = \frac{p(1-f_4)\kappa_4(p+(1-p)f_4)}{f_4^2}. \tag{18}$$

Note that, conditional on $\overline{\mathbb{D}}_{\kappa_4}$, expected leakage is the proportion, p, of arrivals not inspected (Equation (16)) — as would be expected.

Consider now the case in which contamination *is* detected within the κ_4 inspections of Mode 4. The probability that this event occurs is (from above),

$$\mathbb{P}(\mathbb{D}_{\kappa_4}) = 1 - \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) = 1 - (1-p)^{\kappa_4}.$$

The probability that the *first* detection of contamination occurs in the h^{th} inspection of the sequence ($h = 1, 2, \ldots, \kappa_4$), and the associated probability generation function, are provided in Appendix A.1, Equations (A.1)–(A.2), for which each value of H provides the conditional number of consignments inspected. In this case, conditional on $\mathbb{D}_{\kappa_4}$, the number of inspections can take on a distribution of values $(I_4|\mathbb{D}_{\kappa_4}) \in \{1, 2, \ldots, \kappa_4\}$, with probability mass and generating function

$$\mathbb{P}(I_4 = h|\mathbb{D}_{\kappa_4}) = \frac{p(1-p)^{h-1}}{\mathbb{P}(\mathbb{D}_{\kappa_4})} \qquad\qquad \Phi_{I_4|\mathbb{D}_{\kappa_4}}(s) = \frac{ps}{\mathbb{P}(\mathbb{D}_{\kappa_4})}\left(\frac{1-((1-p)s)^{\kappa_4}}{1-(1-p)s}\right) \tag{19}$$

and associated expected value and variance

$$\begin{aligned} \mathbb{E}(I_4|\mathbb{D}_{\kappa_4}) &= \frac{1-(1-p)^{\kappa_4}(\kappa_4 p + 1)}{p\left(1-(1-p)^{\kappa_4}\right)} \\ \mathbb{V}\mathrm{ar}(I_4|\mathbb{D}_{\kappa_4}) &= \frac{(1-p)\left(1+(1-p)^{2\kappa_4}\right) - (1-p)^{\kappa_4}((\kappa_4 p)^2 + 2(1-p))}{p^2\left(1-(1-p)^{\kappa_4}\right)^2}. \end{aligned} \tag{20}$$

Given the conditional number of inspections $(I_4|\mathbb{D}_{\kappa_4})$, associated distributions for the number of consignment arrivals, the number not inspected and the number that leak through the system, can be constructed similarly to those above (results (13)–(18)). It follows that, for the conditional number of arrivals up to and including the I_4^{th} inspection, the associated generating function becomes (see result (13))

$$\begin{aligned} \Phi_{A_4|\mathbb{D}_{\kappa_4}}(s) &= \sum_{I_4|\mathbb{D}_{\kappa_4}} \left(\frac{f_4 s}{1-(1-f_4)s}\right)^{I_4|\mathbb{D}_{\kappa_4}} \mathbb{P}(I_4|\mathbb{D}_{\kappa_4}) \\ &= \Phi_{I_4|\mathbb{D}_{\kappa_4}}\left(\frac{f_4 s}{1-(1-f_4)s}\right) \\ &= \frac{\left(\frac{p f_4 s}{1-(1-f_4)s}\right)}{\mathbb{P}(\mathbb{D}_{\kappa_4})}\left(\frac{1-\left(\frac{(1-p)f_4 s}{1-(1-f_4)s}\right)^{\kappa_4}}{1-\left(\frac{(1-p)f_4 s}{1-(1-f_4)s}\right)}\right). \end{aligned} \tag{21}$$

Similarly, conditional distributions for the number of arriving consignments that are not inspected, and for leakage, have generating functions (respectively)

$$\Phi_{\overline{I}_4|\mathbb{D}_{\kappa_4}}(s) = \Phi_{I_4|\mathbb{D}_{\kappa_4}}\left(\frac{f_4}{1-(1-f_4)s}\right) \tag{22}$$

$$\Phi_{L_4|\mathbb{D}_{\kappa_4}}(s) = \Phi_{I_4|\mathbb{D}_{\kappa_4}}\left(\frac{f_4}{1-(1-f_4)(1-p+ps)}\right) \tag{23}$$

where $\Phi_{I_4|\mathbb{D}_{\kappa_4}}(s)$ is defined in (19). Associated conditional expected values, for arrivals, inspections and leakage, are then

$$\mathbb{E}(A_4|\mathbb{D}_{\kappa_4}) = \frac{\mathbb{E}(I_4|\mathbb{D}_{\kappa_4})}{f_4} = \frac{1-(1-p)^{\kappa_4}(\kappa_4 p+1)}{pf_4\left(1-(1-p)^{\kappa_4}\right)} \tag{24}$$

$$\mathbb{E}(I_4|\mathbb{D}_{\kappa_4}) = \mathbb{E}(I_4|\mathbb{D}_{\kappa_4}) = \frac{1-(1-p)^{\kappa_4}(\kappa_4 p+1)}{p\left(1-(1-p)^{\kappa_4}\right)} \tag{25}$$

$$\mathbb{E}(L_4|\mathbb{D}_{\kappa_4}) = \frac{p(1-f_4)\mathbb{E}(I_4|\mathbb{D}_{\kappa_4})}{f_4} = \frac{(1-f_4)\left(1-(1-p)^{\kappa_4}(\kappa_4 p+1)\right)}{f_4\left(1-(1-p)^{\kappa_4}\right)}. \tag{26}$$

Combining the above results, distributions for the total number of arrivals, total number of inspections and total leakage in Mode 4, have generating functions, respectively,

$$\begin{aligned}\Phi_{A_4}(s) &= \Phi_{A_4|\overline{\mathbb{D}}_{\kappa_4}}(s)\,\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) + \Phi_{A_4|\mathbb{D}_{\kappa_4}}(s)\,\mathbb{P}(\mathbb{D}_{\kappa_4}) \\ &= \left(\frac{(1-p)f_4 s}{1-(1-f_4)s}\right)^{\kappa_4} + \frac{pf_4 s\left(1-\left(\frac{(1-p)f_4 s}{1-(1-f_4)s}\right)^{\kappa_4}\right)}{1-(1-pf_4)s}\end{aligned} \tag{27}$$

$$\begin{aligned}\Phi_{I_4}(s) &= \Phi_{I_4|\overline{\mathbb{D}}_{\kappa_4}}(s)\,\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) + \Phi_{I_4|\mathbb{D}_{\kappa_4}}(s)\,\mathbb{P}(\mathbb{D}_{\kappa_4}) \\ &= \Big((1-p)s\Big)^{\kappa_4} + \frac{ps\left(1-((1-p)s)^{\kappa_4}\right)}{1-(1-p)s}\end{aligned} \tag{28}$$

$$\begin{aligned}\Phi_{L_4}(s) &= \Phi_{L_4|\overline{\mathbb{D}}_{\kappa_4}}(s)\,\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) + \Phi_{L_4|\mathbb{D}_{\kappa_4}}(s)\,\mathbb{P}(\mathbb{D}_{\kappa_4}) \\ &= \left(\frac{f_4(1-p)}{1-(1-f_4)(1-p+ps)}\right)^{\kappa_4} + \frac{f_4\left(1-\left(\frac{f_4(1-p)}{1-(1-f_4)(1-p+ps)}\right)^{\kappa_4}\right)}{1-(1-f_4)s}\end{aligned} \tag{29}$$

The expected number of arrivals, inspections and leakage for Mode 4 then simplify to, respectively,

$$\mathbb{E}(A_4) = \frac{1-(1-p)^{\kappa_4}}{pf_4} \tag{30}$$

$$\mathbb{E}(I_4) = \frac{1-(1-p)^{\kappa_4}}{p} \tag{31}$$

$$\mathbb{E}(L_4) = \frac{(1-f_4)\left(1-(1-p)^{\kappa_4}\right)}{f_4} \tag{32}$$

with associated variance

$$\mathbb{V}\mathrm{ar}(A_4) = \frac{1-pf_4-(1-p)^{2\kappa_4}-p(2\kappa_4-f_4)(1-p)^{\kappa_4}}{(pf_4)^2} \tag{33}$$

$$\mathbb{V}\mathrm{ar}(I_4) = \frac{1-p-(1-p)^{2\kappa_4}-p(2p\kappa_4-1)(1-p)^{\kappa_4}-2p\kappa_4(1-p)^{\kappa_4+1}}{p^2} \tag{34}$$

$$\mathbb{V}\mathrm{ar}(L_4) = \frac{(1-f_4)\left(1-(1-f_4)(1-p)^{2\kappa_4}-\left(f_4+(1-f)2p\kappa_4\right)(1-p)^{\kappa_4}\right)}{f_4^2}. \tag{35}$$

2.2 Distributions and statistics for each mode — with imperfect detection

Commonly, consignments that arrive at the Australian border are sampled, with samples tested for contamination. This is not a perfect test. It was shown in [1] that taking test-sensitivity of this nature into account can alter system design strategies for CSP-1 in fundamental ways. We thus incorporate sensitivity into the above results for each mode of CSP-3 (and CSP-2). Our analytical expressions for the relevant distributions and statistics enable a straightforward means of understanding the influence of system controls on risk, and thereby offer a powerful guide for robust CSP-2 and CSP-3 design principles when detection is imperfect.

Let δ, with $0 < \delta \leq 1$, be the probability that contamination is detected when present in a consignment inspected at the border. We exclude the case of $\delta = 0$, although it is a trivial special case. As above, we define $\overline{\mathbb{D}}_\kappa$ as the event that there are no detections in a sequence of κ inspections, and define $\mathbb{D}_\kappa$ as the complementary event. Taking imperfect detection into account, it then follows that, conditional on p,

$$\mathbb{P}(\overline{\mathbb{D}}_\kappa) = (1-\delta p)^\kappa \qquad \mathbb{P}(\mathbb{D}_\kappa) = 1-(1-\delta p)^\kappa. \tag{36}$$

As above, for each mode j ($j = 1, 2, 3$ and 4), we define random variables $A_{j\delta}$, $I_{j\delta}$ and $L_{j\delta}$ for the number of consignment arrivals, the number of consignment inspections, and the number of consignments that leak through the system, respectively — during a single pass through mode j with δ identifying that the inspection process may be imperfect.

2.2.1 Mode 1 — Census Mode

Distributions and statistics for Mode 1, when detection is assumed imperfect, were formulated in [1]. Using current notation, the probability generating function, expected value and variance for the number of arriving consignments can be expressed

$$\Phi_{A_{1\delta}}(s) = \frac{\big((1-\delta p)s\big)^{\kappa_1}(1-(1-\delta p)s)}{1-s+\delta ps\,((1-\delta p)s)^{\kappa_1}} \tag{37}$$

$$\mathbb{E}(A_{1\delta}) = \frac{(1-\delta p)^{-\kappa_1}-1}{\delta p} \tag{38}$$

$$\mathbb{V}\mathrm{ar}(A_{1\delta}) = \frac{(1-\delta p)^{-2\kappa_1}-\delta p(2\kappa_1+1)(1-\delta p)^{-\kappa_1}-(1-\delta p)}{(\delta p)^2}. \tag{39}$$

The number of inspections in Mode 1 is given by the number of arrivals because all arriving consignments are inspected. When detection is imperfect, however, unlike for the perfect case, leakage *can* occur because contamination may escape detection during inspection. The associated probability generating function, expected value and variance for leakage through Mode 1 are [1]

$$\Phi_{L_{1\delta}}(s) = \frac{\big(1-p\,(1-(1-\delta)s)\big)^{\kappa_1}(1-(1-\delta)s)}{\delta\,\big(1-p\,(1-(1-\delta)s)\big)^{\kappa_1}-(1-\delta)(s-1)} \tag{40}$$

$$\mathbb{E}(L_{1\delta}) = \frac{(1-\delta)\,\big((1-\delta p)^{-\kappa_1}-1\big)}{\delta} \tag{41}$$

$$\begin{aligned}&\mathbb{V}\mathrm{ar}(L_{1\delta})\\ &= \frac{(1-\delta)\,\big((1-\delta)(1-\delta p)^{-2\kappa_1}-2(1+(\kappa_1-1)\delta p)(1-\delta)(1-\delta p)^{-\kappa_1-1}+(2-\delta)(1-\delta p)^{-\kappa_1}\big)}{\delta^2}.\end{aligned} \tag{42}$$

2.2.2 Mode 2 — Monitoring Mode

Distributions and statistics for Mode 2, when detection is assumed imperfect, were formulated in [1]. Using current notation, the probability generating function, expected value and variance for the number of arriving consignments can be expressed

$$\Phi_{A_{2\delta}}(s) = \frac{\delta pf_2 s}{1-(1-\delta pf_2)s}, \qquad \mathbb{E}(A_{2\delta}) = \frac{1}{\delta pf_2}, \qquad \mathbb{V}\mathrm{ar}(A_{2\delta}) = \frac{1-\delta pf_2}{(\delta pf_2)^2}. \tag{43}$$

The associated probability generating function, expected value and variance for the number of inspected consignments are [1]

$$\Phi_{I_{2\delta}}(s) = \frac{\delta p s}{1-(1-\delta p)s}, \qquad \mathbb{E}(I_{2\delta}) = \frac{1}{\delta p}, \qquad \mathbb{V}\mathrm{ar}(I_{2\delta}) = \frac{1-\delta p}{(\delta p)^2}. \tag{44}$$

When detection is imperfect, leakage can occur when consignments are not inspected (as in the perfect case), but also during the inspection process. The probability generating function, expected value and variance for total leakage through Mode 2 are [1]

$$\Phi_{L_{2\delta}}(s) = \frac{\delta p f_2(1-\delta p f_2)}{1-\delta p f_2 - \big(1-pf_2+pf_2(1-\delta)s\big)\big(1-\delta p f - p(1-f_2)+p(1-f_2)s\big)} \tag{45}$$

$$\mathbb{E}(L_{2\delta}) = \frac{(1-\delta f_2)}{\delta f_2} \tag{46}$$

$$\mathbb{V}\mathrm{ar}(L_{2\delta}) = \frac{1-\delta f_2}{(\delta f_2)^2} + 2\left(\frac{p(1-\delta)(1-f_2)}{\delta(1-\delta p f_2)}\right). \tag{47}$$

2.2.3 Mode 3

Mode 3 was not considered in [1], so we provide full details for the inclusion of inspection sensitivity into the results of Section 2.1.3. First note that, in Mode 3 with imperfect detection, leakage can occur when contaminated consignments are inspected but contamination is not detected — in contrast to the case that assumes perfect detection, where no leakage could occur.

For arrivals in Mode 3, note that δp is the probability a randomly selected consignment is both contaminated and detected when inspected. The probability generating function, expected value and variance for the number of arrivals with inspection sensitivity included are then (see Section 2.1.3)

$$\Phi_{A_{3\delta}}(s) = s^{\kappa_3}\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3}) + \Phi_{H|\mathbb{D}_{\kappa_3}}(s)\mathbb{P}(\mathbb{D}_{\kappa_3}) = \Big((1-\delta p)s\Big)^{\kappa_3} + \frac{\delta p s\left(1-\big((1-\delta p)s\big)^{\kappa_3}\right)}{(1-(1-\delta p)s)} \tag{48}$$

$$\mathbb{E}(A_{3\delta}) = \frac{1-(1-\delta p)^{\kappa_3}}{\delta p} \tag{49}$$

$$\mathbb{V}\mathrm{ar}(A_{3\delta}) = \frac{(1-\delta p)\left(1-(1-\delta p)^{2\kappa_3-1} - 2p\kappa_3(1-\delta p)^{\kappa_3} + \delta p(1-2\delta p\kappa_3)(1-\delta p)^{\kappa_3-1}\right)}{(\delta p)^2}. \tag{50}$$

The number of inspections in Mode 3 is given by the number of arrivals because all arriving consignments are inspected.

Leakage can occur in Mode 3 when an imperfect inspection fails to detect contamination that is present. In the case contamination is detected in the sequence of inspections (event $\mathbb{D}_{\kappa_3}$), leakage can only occur during the inspections *before* that in which it is detected. Alternatively, in the case contamination is not detected (event $\overline{\mathbb{D}}_{\kappa_3}$), leakage can occur during any of κ_3 inspections. A distribution for the number of arriving-and-inspected consignments in which no contamination is detected (interim random variable $A^*_{3\delta}$), can be determined from (48) above — with probability generating function,

$$\Phi_{A^*_{3\delta}}(s) = s^{\kappa_3}\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3}) + \frac{\Phi_{H|\mathbb{D}_{\kappa_3}}(s)}{s}\mathbb{P}(\mathbb{D}_{\kappa_3}) = \Big((1-\delta p)s\Big)^{\kappa_3} + \frac{\delta p\left(1-\big((1-\delta p)s\big)^{\kappa_3}\right)}{1-(1-\delta p)s},$$

where $\Phi_{H|\mathbb{D}_{\kappa_3}}(s)$ is defined in Appendix A (Equation A.1). Since leakage is the number of consignments inspected, contaminated and not-detected, the distribution for leakage then has probability generating function

$$
\begin{aligned}
\Phi_{L_{3\delta}}(s) &= \left((1-\delta p) \left(1 - \frac{(1-\delta)p}{1-\delta p} + \left(\frac{(1-\delta)p}{1-\delta p} \right) s \right) \right)^{\kappa_3} \\
&\qquad + \frac{\delta p \left(1 - \left((1-\delta p) \left(1 - \frac{(1-\delta)p}{1-\delta p} + \left(\frac{(1-\delta)p}{1-\delta p} \right) s \right) \right)^{\kappa_3} \right)}{1 - (1-\delta p) \left(1 - \frac{(1-\delta)p}{1-\delta p} + \left(\frac{(1-\delta)p}{1-\delta p} \right) s \right)} \\
&= \left(1 - p + (1-\delta)ps \right)^{\kappa_3} + \frac{\delta \left(1 - \left(1 - p + (1-\delta)ps \right)^{\kappa_3} \right)}{1 - (1-\delta)s} \qquad (51)
\end{aligned}
$$

with associated expected value and variance

$$
\begin{aligned}
\mathbb{E}(L_{3\delta}) &= \frac{(1-\delta)\left(1 - (1-\delta p)^{\kappa_3}\right)}{\delta} \qquad (52) \\
\mathbb{V}\mathrm{ar}(L_{3\delta}) &= \frac{(1-\delta)}{\delta^2} \left(\left(1 + (1-\delta)(1-\delta p)^{\kappa_3} \right) \left(1 - (1-\delta p)^{\kappa_3} \right) - 2\delta p \kappa_3 (1-\delta)(1-\delta p)^{\kappa_3 - 1} \right).
\end{aligned}
$$

Note that, when $\delta = 1$ and perfect detection is assumed, the above results reduce to those in Section 2.1.3 — as would be expected.

2.2.4 Mode 4

Mode 4 was not considered in [1] and we thus provide full details for the inclusion of inspection sensitivity into the results of Section 2.1.4. First note that, in Mode 4 with imperfect detection, leakage can occur in two distinct ways: when consignments are not inspected (as in the case with perfect detection); and when consignments are inspected but no contamination is detected.

First consider the case in which there are no detections in κ_4 sequential inspections — event $\overline{\mathbb{D}}_{\kappa_4}$. The conditional number of inspections is κ_4, and the conditional number of arrivals, and number of arrivals-not-inspected have generating functions, expected values and variances as given in Section 2.1.4 — equations (13)–(16). Conditional on $\overline{\mathbb{D}}_{\kappa_4}$, these random variables are independent from δ.

When detection is imperfect, and conditional on $\overline{\mathbb{D}}_{\kappa_4}$, the first component of leakage when consignments are not inspected, denoted $L_{4\delta_1}$, has probability generating function (see results (17)–(18))

$$
\Phi_{L_{4\delta_1}|\overline{\mathbb{D}}_{\kappa_4}}(s) = \left(\frac{f_4}{1 - (1-f_4)(1-p+ps)} \right)^{\kappa_4}. \qquad (53)
$$

There is a second component of leakage — denoted $L_{4\delta_2}$. There are κ_4 inspections with no detections, but undertaken using an imperfect test. This component of leakage, conditional on $\overline{\mathbb{D}}_{\kappa_4}$, has probability generating function (see result (51))

$$
\Phi_{L_{4\delta_2}|\overline{\mathbb{D}}_{\kappa_4}}(s) = \left(1 - \frac{(1-\delta)p}{1-\delta p} + \left(\frac{(1-\delta)p}{1-\delta p} \right) s \right)^{\kappa_4} \qquad (54)
$$

Both components are conditional on $\overline{\mathbb{D}}_{\kappa_4}$ (κ_4 inspections), so that the total conditional leakage has generating function

$$
\begin{aligned}
\Phi_{(L_{4\delta_1} + L_{4\delta_2})|\overline{\mathbb{D}}_{\kappa_4}}(s) &= \left(\frac{f_4 \left(1 - p + (1-\delta)ps \right)}{\left(1 - (1-f_4)(1-p+ps) \right)(1-\delta p)} \right)^{\kappa_4} \\
&= \frac{1}{\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})} \left(\frac{f_4 \left(1 - p + (1-\delta)ps \right)}{1 - (1-f_4)(1-p+ps)} \right)^{\kappa_4}. \qquad (55)
\end{aligned}
$$

Alternatively, consider the case in which contamination *is* detected within the κ_4 inspections of Mode 4 — that is, conditional on event $\mathbb{D}_{\kappa_4}$. Analogous to results (19)–(20) and the associated discussion, the conditional number of inspections has generating function, expected value and variance

$$\Phi_{I_{4\delta}|\mathbb{D}_{\kappa_4}}(s) = \frac{\delta ps}{\mathbb{P}(\mathbb{D}_{\kappa_4})}\left(\frac{1-((1-\delta p)s)^{\kappa_4}}{1-(1-\delta p)s}\right) \tag{56}$$

$$\mathbb{E}(I_{4\delta}|\mathbb{D}_{\kappa_4}) = \frac{1-(1-p)^{\kappa_4}\,(\kappa_4\delta p+1)}{\delta p\left(1-(1-\delta p)^{\kappa_4}\right)} \tag{57}$$

$$\mathbb{V}\mathrm{ar}(I_{4\delta}|\mathbb{D}_{\kappa_4}) = \frac{(1-\delta p)\left(1+(1-\delta p)^{2\kappa_4}\right)-((\kappa_4\delta p)^2-2\delta p+2)(1-p)^{\kappa_4}}{(\delta p)^2\left(1-(1-\delta p)^{\kappa_4}\right)^2}. \tag{58}$$

Analogous to (21), for the conditional number of arrivals up to and including the inspection in which contamination is detected,

$$\Phi_{A_{4\delta}|\mathbb{D}_{\kappa_4}}(s) \;=\; \Phi_{I_{4\delta}|\mathbb{D}_{\kappa_4}}\!\left(\frac{f_4 s}{1-(1-f_4)s}\right) \;=\; \frac{\left(\frac{\delta p f_4 s}{1-(1-f_4)s}\right)}{\mathbb{P}(\mathbb{D}_{\kappa_4})}\left(\frac{1-\left(\frac{(1-\delta p)f_4 s}{1-(1-f_4)s}\right)^{\kappa_4}}{1-\left(\frac{(1-\delta p)f_4 s}{1-(1-f_4)s}\right)}\right). \tag{59}$$

Leakage, conditional on $\mathbb{D}_{\kappa_4}$, can occur in two distinct ways. When detection is imperfect, the first component of conditional leakage occurs when consignments are not inspected and has probability generating function (analogous to (23))

$$\Phi_{L_{4\delta_1}|\mathbb{D}_{\kappa_4}}(s) = \Phi_{I_{4\delta}|\mathbb{D}_{\kappa_4}}\left(\frac{f_4}{1-(1-f_4)(1-p+ps)}\right). \tag{60}$$

For the second component of leakage, there is a fixed number of inspections *before* that in which contamination is detected — in any of which leakage could occur because inspection is imperfect. The distribution for this component has generating function (analogous to Mode 3 — results (51)–(52))

$$\Phi_{L_{4\delta_2}|\mathbb{D}_{\kappa_4}}(s) = \left(\Phi_{I_{4\delta}|\mathbb{D}_{\kappa_4}}\!\left(1-\frac{(1-\delta)p}{1-\delta p}+\left(\frac{(1-\delta)p}{1-\delta p}\right)s\right)\right)\Bigg/\left(1-\frac{(1-\delta)p}{1-\delta p}+\left(\frac{(1-\delta)p}{1-\delta p}\right)s\right). \tag{61}$$

Both components depend on the conditional number of inspections $(I_{4\delta}|\mathbb{D}_{\kappa_4})$ but are conditionally independent, so that total conditional leakage has probability generating function

$$\begin{aligned}
&\Phi_{(L_4\delta_1+L_{4\delta_2})|\mathbb{D}_{\kappa_4}}(s)\\
&=\Phi_{I_{4\delta}|\mathbb{D}_{\kappa_4}}\!\left(\left(1-\frac{(1-\delta)p}{1-\delta p}+\frac{(1-\delta)ps}{1-\delta p}\right)\left(\frac{f_4}{1-(1-f_4)(1-p+ps)}\right)\right)\Bigg/\left(1-\frac{(1-\delta)p}{1-\delta p}+\frac{(1-\delta)ps}{1-\delta p}\right)\\
&=\frac{\delta f_4\left(1-\left(\frac{f_4(1-p+(1-\delta)ps)}{1-(1-f_4)(1-p+ps)}\right)^{\kappa_4}\right)}{\mathbb{P}(\mathbb{D}_{\kappa_4})\left(1-(1-\delta f_4)s\right)}.
\end{aligned} \tag{62}$$

Finally, we combine the above results to formulate distributions, expected values and variances for the total number of arrivals, inspections and leakage in Mode 4.

Combining the conditional distributions for arrivals (results (13) and (59)), the number of arrivals in Mode 4 has probability generating function,

$$\begin{aligned}
\Phi_{A_{4\delta}}(s) &= \left(\frac{f_4 s}{1-(1-f_4)s}\right)^{\kappa_4}\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) \;+\; \Phi_{I_{4\delta}|\mathbb{D}_{\kappa_4}}\!\left(\frac{f_4 s}{1-(1-f_4)s}\right)\mathbb{P}(\mathbb{D}_{\kappa_4})\\
&= \left(\frac{(1-\delta p)f_4 s}{1-(1-f_4)s}\right)^{\kappa_4} + \delta p f_4 s\left(\frac{1-\left(\frac{(1-\delta p)f_4 s}{1-(1-f_4)s}\right)^{\kappa_4}}{1-(1-\delta p f_4)s}\right)
\end{aligned} \tag{63}$$

with associated expected value and variance,

$$\mathbb{E}(A_{4\delta}) = \frac{1-(1-\delta p)^{\kappa_4}}{\delta p f_4} \tag{64}$$

$$\mathbb{V}\mathrm{ar}(A_{4\delta}) = \frac{1-\delta p f_4 + \delta p(f_4 - 2\kappa_4)(1-\delta p)^{\kappa_4} - (1-\delta p)^{2\kappa_4}}{(\delta p f_4)^2}. \tag{65}$$

Note that, when $f = 1$ and all arriving consignments are inspected, generating function (63) reduces to (48), and when, additionally, $\delta = 1$ and perfect detection is assumed, it reduces further to (10) — as would be expected.

Combining the conditional distributions for inspections (see (56)), the number of inspections in Mode 4 has probability generating function,

$$\begin{aligned}\Phi_{I_4\delta}(s) &= s^{\kappa_4}\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) + \frac{\delta p s}{\mathbb{P}(\mathbb{D}_{\kappa_4})}\left(\frac{1-((1-\delta p)s)^{\kappa_4}}{1-(1-\delta p)s}\right)\mathbb{P}(\mathbb{D}_{\kappa_4}) \\ &= \Big((1-\delta p)s\Big)^{\kappa_4} + \frac{\delta p s\left(1-\left((1-\delta p)s\right)^{\kappa_4}\right)}{1-(1-\delta p)s}\end{aligned} \tag{66}$$

with associated expected value and variance

$$\mathbb{E}(I_{4\delta}) = \frac{1-(1-\delta p)^{\kappa_4}}{\delta p} \tag{67}$$

$$\mathbb{V}\mathrm{ar}(I_{4\delta}) = \frac{1-\delta p - 2\kappa_4\delta p(1-\delta p)^{\kappa_4+1} + \delta p(1-2\delta p\kappa_4)(1-\delta p)^{\kappa_4} - (1-\delta p)^{2\kappa_4}}{(\delta p)^2}. \tag{68}$$

Similarly, combining the results for conditional leakage (results (55) and (62)), the number of consignments that leak through Mode 4 has probability generating function,

$$\begin{aligned}\Phi_{L_{4\delta}}(s) &= \Big(\Phi_{(L_{4\delta_1}+L_{\delta_2})|\overline{\mathbb{D}}_{\kappa_4}}(s)\Big)\,\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) + \Big(\Phi_{(L_{4\delta_1}+L_{\delta_2})|\mathbb{D}_{\kappa_4}}(s)\Big)\,\mathbb{P}(\mathbb{D}_{\kappa_4}) \\ &= \left(\frac{f_4\left(1-p+(1-\delta)ps\right)}{1-(1-f_4)(1-p+ps)}\right)^{\kappa_4} + \frac{\delta f_4\left(1-\left(\frac{f_4(1-p+(1-\delta)ps)}{1-(1-f_4)(1-p+ps)}\right)^{\kappa_4}\right)}{1-(1-\delta f_4)s}\end{aligned} \tag{69}$$

with associated expected value and variance

$$\mathbb{E}(L_{4\delta}) = \frac{(1-\delta f_4)\left(1-(1-\delta p)^{\kappa_4}\right)}{\delta f_4} \tag{70}$$

$$\mathbb{V}\mathrm{ar}(L_{4\delta}) = \frac{(1-\delta f_4)\left((1+(1-\delta f)(1-\delta p)^{\kappa_4})\left(1-(1-\delta p)^{\kappa_4}\right) - 2\delta p\kappa_4\left(1-\delta(p+(1-p)f_4)\right)(1-\delta p)^{\kappa_4-1}\right)}{(\delta f_4)^2}. \tag{71}$$

Note that, when $f_4 = 1$ and all arriving consignments are inspected, generating function (69) reduces to (51). And when $\delta = 1$ and perfect detection is assumed, it reduces to (27) — as would be expected.

3 Stochastic model for a full cycle of CSP-2 and CSP-3

The above results for each mode can be combined to provide simple algebraic expressions for the distributions, expected values and variances associated with the total number of arrivals, inspections and leakage — over a full cycle of the CSP-2 and CSP-3 systems. As discussed, the biosecurity process is assumed to start in Mode 1 (Census Mode), and a full cycle is considered complete when an importer is returned to Mode 1 after transitioning through Mode 2. We highlight that, exactly one pass through Mode 1 occurs in a cycle of CSP-2 or CSP-3 and, before transitioning back to Mode 1, many passes through Modes 2, 3 (in the case of CSP-3) and 4 could occur.

3.1 Expected number of arrivals, inspections and leakage in a cycle

We first construct expressions for expected values, where the formulation makes use of conditional independence between mode-level distributions — although they can also be derived directly from the generating functions formulated below (Section 3.2).

Let Z denote a generic random variable for the 'total number' (of arrivals, of inspections or of leakage in a cycle), where random variables M_1, M_2, M_3 and M_4 denote generic contributions from Modes 1–4, respectively, and where, for mode j, with $j = 1, 2, 3, 4$, and for perfect and imperfect detection, respectively,

$$\mathbb{P}(\overline{\mathbb{D}}_{\kappa_j}) = (1-p)^{\kappa_j}, \qquad \mathbb{P}(\overline{\mathbb{D}}_{\kappa_j}) = (1-\delta p)^{\kappa_j}.$$

For CSP-2, the expected value can be expressed (see Appendix B)

$$\begin{aligned}
\mathbb{E}(Z^{(\text{CSP2})}) &= \mathbb{E}(M_1) + \mathbb{E}(M_2) + \mathbb{P}(\mathbb{D}_{\kappa_4})\mathbb{E}(M_4|\mathbb{D}_{\kappa_4}) \\
&\quad + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) \Big[\kappa_4 + \mathbb{E}(M_2) + \mathbb{P}(\mathbb{D}_{\kappa_4})\mathbb{E}(M_4|\mathbb{D}_{\kappa_4}) \\
&\quad + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) \Big[\kappa_4 + \mathbb{E}(M_2) + \mathbb{P}(\mathbb{D}_{\kappa_4})\mathbb{E}(M_4|\mathbb{D}_{\kappa_4}) \\
&\quad + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) \Big[\cdots\cdots\Big]\Big]\Big] \\
&= \mathbb{E}(M_1) + \mathbb{E}(M_2)\Big(1 + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) + \big(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\big)^2 + \big(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\big)^3 + \cdots\Big) \\
&\quad + \mathbb{E}(M_4)\left(1 + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) + \left(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\right)^2 + \left(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\right)^3 + \cdots\right) \\
&= \mathbb{E}(M_1) + \frac{\mathbb{E}(M_2) + \mathbb{E}(M_4)}{1 - \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})}.
\end{aligned} \tag{72}$$

The total expected number of arrivals, inspections and leakage in a complete cycle of CSP-2, can then be determined by simply substituting A_j, I_j or L_j for M_j in expression (72), where expressions for all expected values ($\mathbb{E}(A_j)$, $\mathbb{E}(I_j)$ or $\mathbb{E}(L_j)$) are provided in Section 4.

As an example, for the total expected number of arrivals in a complete cycle of CSP-2 (notated $\mathbb{E}(A^{(\text{CSP2})})$), substituting from results (83)) in Section 4 and assuming $\delta = 1$ (perfect detection), $f_2 = f_4 = f$ and $\kappa_4 = \kappa$, function (72) reduces to

$$\mathbb{E}(A^{(\text{CSP2})}) = \frac{1-(1-p)^{\kappa_1}}{p(1-p)^{\kappa_1}} + \frac{2-(1-p)^{\kappa}}{pf\left(1-(1-p)^{\kappa}\right)}. \tag{73}$$

Similarly, under the same assumptions and substitution from results (84) and (85) in Section 4, the expected number of inspections and leakage, respectively, reduce to

$$\mathbb{E}(I^{(\text{CSP2})}) = \frac{1}{p}\left(\frac{1}{(1-p)^{\kappa_1}} + \frac{1}{1-(1-p)^{\kappa}}\right) \tag{74}$$

$$\mathbb{E}(L^{(\text{CSP2})}) = \frac{(1-f)\left(2-(1-p)^{\kappa}\right)}{f\left(1-(1-p)^{\kappa}\right)}. \tag{75}$$

For CSP-3, the expected value can be expressed

$$
\begin{aligned}
\mathbb{E}(Z^{\text{(CSP3)}}) &= \mathbb{E}(M_1) + \mathbb{E}(M_2) + \mathbb{P}(\mathbb{D}_{\kappa_3})\mathbb{E}(M_3|\mathbb{D}_{\kappa_3}) \\
&\quad + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\Big[\kappa_3 + \mathbb{P}(\mathbb{D}_{\kappa_4})\mathbb{E}(M_4|\mathbb{D}_{\kappa_4}) + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\Big[\kappa_4 + \mathbb{E}(M_2) + \mathbb{P}(\mathbb{D}_{\kappa_3})\mathbb{E}(M_3|\mathbb{D}_{\kappa_3}) \\
&\quad + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\Big[\kappa_3 + \mathbb{P}(\mathbb{D}_{\kappa_4})\mathbb{E}(M_4|\mathbb{D}_{\kappa_4}) + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\Big[\kappa_4 + \mathbb{E}(M_2) + \mathbb{P}(\mathbb{D}_{\kappa_3})\mathbb{E}(M_3|\mathbb{D}_{\kappa_3}) \\
&\quad + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\Big[\cdots\cdots\Big]\Big]\Big]\Big]\Big] \\
&= \mathbb{E}(M_1) + \mathbb{E}(M_2)\Big(1 + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) + \Big(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\Big)^2 + \Big(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\Big)^3 + \cdots\Big) \\
&\quad + \mathbb{E}(M_3)\Big(1 + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) + \Big(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\Big)^2 + \Big(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\Big)^3 + \cdots\Big) \\
&\quad + \mathbb{E}(M_4)\Big(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3}) + \Big(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\Big)^2\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}) + \Big(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\Big)^3\Big(\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\Big)^2 + \cdots\Big) \\
&= \mathbb{E}(M_1) + \frac{\mathbb{E}(M_2) + \mathbb{E}(M_3) + \mathbb{E}(M_4)\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})}{1 - \mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})}\,. \qquad (76)
\end{aligned}
$$

As above, the total expected number of arrivals, inspections and leakage in a complete cycle of CSP-3, can then be determined by simply substituting $\mathbb{E}(A_j)$, $\mathbb{E}(I_j)$ or $\mathbb{E}(L_j)$ (Section 4) for $\mathbb{E}(M_j)$ in expression (76).

As an example, for the total expected number of arrivals in a complete cycle of CSP-3 (notated $\mathbb{E}(A^{\text{(CSP3)}})$), substituting from results (83) in Section 4 and assuming $\delta = 1$ (perfect detection), $f_2 = f_4 = f$ and $\kappa_3 = \kappa_4 = \kappa$, function (76) reduces to

$$
\mathbb{E}(A^{\text{(CSP3)}}) = \frac{1-(1-p)^{\kappa_1}}{p(1-p)^{\kappa_1}} + \frac{(1+f) + (1-f)(1-p)^{\kappa} - (1-p)^{2\kappa}}{pf\left(1-(1-p)^{2\kappa}\right)}\,. \qquad (77)
$$

Similarly, under the same assumptions and substitution from results (84) and (85) in Section 4 , the expected number of inspections and leakage, respectively, reduce to

$$
\mathbb{E}(I^{\text{(CSP3)}}) = \frac{1}{p}\left(\frac{1}{(1-p)^{\kappa_1}} + \frac{1}{1-(1-p)^{2\kappa}}\right) \qquad (78)
$$

$$
\mathbb{E}(L^{\text{(CSP3)}}) = \frac{(1-f)\left(1 + (1-p)^{\kappa} - (1-p)^{2\kappa}\right)}{f\left(1-(1-p)^{2\kappa}\right)}\,. \qquad (79)
$$

See Appendix B for a comparison between results (72) and (76), expected value ratios, and equivalent CSP-2 and CSP-3 expected-values provided in [5].

3.2 Distributions for the number of arrivals, inspections and leakage in a cycle

As above, distributions for the total expected number of arrivals, inspections and leakage in a full cycle of CSP2 and CSP-3 can be constructed. Using the above notation, the 'generic' probability generating functions for the distributions can be expressed (Appendix C)

$$
\Phi_Z^{\text{(CSP2)}}(s) = \Phi_{M_1}(s)\left(\frac{\Phi_{M_2}(s)\Psi_{4a}}{1-\Phi_{M_2}(s)\Psi_{4b}}\right) \qquad \text{CSP-2} \qquad (80\text{a})
$$

$$
\Phi_Z^{\text{(CSP3)}}(s) = \Phi_{M_1}(s)\left(\frac{\Phi_{M_2}(s)\Psi_{M_3a} + \Phi_{M_2}(s)\Psi_{M_3b}\Psi_{M_4a}}{1-\Phi_{M_2}(s)\Psi_{M_3b}\Psi_{M_4b}}\right) \qquad \text{CSP-3} \qquad (80\text{b})
$$

where

$$
\begin{aligned}
\Psi_{M_3 a} &= \Phi_{M_3|\mathbb{D}_{\kappa_3}}(s)\,\mathbb{P}(\mathbb{D}_{\kappa_3}) \\
\Psi_{M_3 b} &= \Phi_{M_3|\overline{\mathbb{D}}_{\kappa_3}}(s)\,\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3}) \\
\Psi_{M_4 a} &= \Phi_{M_4|\mathbb{D}_{\kappa_4}}(s)\,\mathbb{P}(\mathbb{D}_{\kappa_4}) \\
\Psi_{M_4 b} &= \Phi_{M_4|\overline{\mathbb{D}}_{\kappa_4}}(s)\,\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}).
\end{aligned}
\tag{81}
$$

As an example, for the total number of consignment arrivals in a complete cycle of CSP-3 (notated $\Phi_A^{(\mathrm{CSP3})}(s)$), substituting from Section 2.1 and assuming $\delta = 1$ (perfect detection), $f_2 = f_4 = f$ and $\kappa_3 = \kappa_4 = \kappa$, function (80b) reduces to

$$
\Phi_A^{(\mathrm{CSP3})}(s) = \frac{\left(\frac{((1-p)s)^{\kappa_1}\left(1-(1-p)s\right)}{1-s+ps(1-((1-p)s)^{\kappa_1})}\right) f p^2 s^2 \left(\frac{1-((1-p)s)^{\kappa}}{1-(1-p)s} + \frac{f((1-p)s)^{\kappa}\left(1-\left(\frac{fs(1-p)}{1-s(1-f)}\right)^{\kappa}\right)}{1-(1-fp)s}\right)}{1 - s\left((1-pf) + pf\left(\frac{s^2 f(1-p)^2}{1-(1-f)s}\right)^{\kappa}\right)}. \tag{82}
$$

The expected value and variance are straightforward to determine from this function (Appendix C). These results can appear complicated, but they are straightforward to construct from simple component expressions, they simplify in many cases, and the associated statistics are easily calculated.

Finally, results (72), (76) and (80) do not depend on a perfect-detection process and hold for distributions and statistics that take inspection sensitivity into account. Expressions for the relevant component expected-values are summarised in Section 4, and the associated distributions are provided in Section 2.2. For each mode, they can be substituted for M_j in results (72), (76) and (80). As discussed above, we reiterate that when imperfect detection is assumed, $\mathbb{P}(\overline{\mathbb{D}}_{\kappa_j}) = (1-\delta p)^{\kappa_j}$.

4 Summary of expected-value results

The above formulations for expected values in a single pass through each mode, and in a single cycle through CSP-2 and CSP-3, are summarised below. As above, a single pass through a mode begins when an importer transitions to the mode, and ends when the importer transitions from that mode. And a single CSP-2 or CSP-3 cycle is assumed to begin when an importer enters Mode 1, and ends when the importer returns to Mode 1 after transitioning through other modes.

Mode-level: In the case detection is assumed perfect ($\delta = 1$)

$$\mathbb{E}(A_1) = \frac{(1-p)^{-\kappa_1} - 1}{p} \qquad \text{expected arrivals in Mode 1 (eq. (3))} \tag{83a}$$

$$\mathbb{E}(A_2) = \frac{1}{p f_2} \qquad \text{expected arrivals in Mode 2 (eq. (6))} \tag{83b}$$

$$\mathbb{E}(A_3) = \frac{1-(1-p)^{\kappa_3}}{p} \qquad \text{expected arrivals in Mode 3 (eq. (11))} \tag{83c}$$

$$\mathbb{E}(A_4) = \frac{1-(1-p)^{\kappa_4}}{p f_4} \qquad \text{expected arrivals in Mode 4 (eq. (30))} \tag{83d}$$

$$\mathbb{E}(I_1) = \frac{(1-p)^{-\kappa_1} - 1}{p} \qquad \text{expected inspections in Mode 1 (eq. (3))} \tag{84a}$$

$$\mathbb{E}(I_2) = \frac{1}{p} \qquad \text{expected inspections in Mode 2 (eq. (7))} \tag{84b}$$

$$\mathbb{E}(I_3) = \frac{1 - (1-p)^{\kappa_3}}{p} \qquad \text{expected inspections in Mode 3 (eq. (11))} \tag{84c}$$

$$\mathbb{E}(I_4) = \frac{1 - (1-p)^{\kappa_4}}{p} \qquad \text{expected inspections in Mode 4 (eq. (31))} \tag{84d}$$

$$\mathbb{E}(L_1) = 0 \qquad \text{expected leakage through Mode 1} \tag{85a}$$

$$\mathbb{E}(L_2) = \frac{1 - f_2}{f_2} \qquad \text{expected leakage through Mode 2 (eq. (8))} \tag{85b}$$

$$\mathbb{E}(L_3) = 0 \qquad \text{expected leakage through Mode 3} \tag{85c}$$

$$\mathbb{E}(L_4) = \frac{(1 - f_4)\,(1 - (1-p)^{\kappa_4})}{f_4} \qquad \text{expected leakage through Mode 4 (eq. (32))} \tag{85d}$$

Mode-level: In the case detection is assumed perfect or imperfect ($0 < \delta \leq 1$)

$$\mathbb{E}(A_{1\delta}) = \frac{(1-\delta p)^{-\kappa_1} - 1}{\delta p} \qquad \text{expected arrivals in Mode 1 (eq. (38))} \tag{86a}$$

$$\mathbb{E}(A_{2\delta}) = \frac{1}{\delta p f_2} \qquad \text{expected arrivals in Mode 2 (eq. (43))} \tag{86b}$$

$$\mathbb{E}(A_{3\delta}) = \frac{1 - (1-\delta p)^{\kappa_3}}{\delta p} \qquad \text{expected arrivals in Mode 3 (eq. (48))} \tag{86c}$$

$$\mathbb{E}(A_{4\delta}) = \frac{1 - (1-\delta p)^{\kappa_4}}{\delta p f_4} \qquad \text{expected arrivals in Mode 4 (eq. (64))} \tag{86d}$$

$$\mathbb{E}(I_{1\delta}) = \frac{(1-\delta p)^{-\kappa_1} - 1}{\delta p} \qquad \text{expected inspections in Mode 1 (eq. (38))} \tag{87a}$$

$$\mathbb{E}(I_{2\delta}) = \frac{1}{\delta p} \qquad \text{expected inspections in Mode 2 (eq. (44))} \tag{87b}$$

$$\mathbb{E}(I_{3\delta}) = \frac{1 - (1-\delta p)^{\kappa_3}}{\delta p} \qquad \text{expected inspections in Mode 3 (eq. (48))} \tag{87c}$$

$$\mathbb{E}(I_{4\delta}) = \frac{1 - (1-\delta p)^{\kappa_4}}{\delta p} \qquad \text{expected inspections in Mode 4 (eq. (67))} \tag{87d}$$

$$\mathbb{E}(L_{1\delta}) = \frac{(1-\delta)\left((1-\delta p)^{-\kappa_1} - 1\right)}{\delta} \qquad \text{expected leakage through Mode 1 (eq. (41))} \tag{88a}$$

$$\mathbb{E}(L_{2\delta}) = \frac{(1 - \delta f_2)}{\delta f_2} \qquad \text{expected leakage through Mode 2 (eq. (41))} \tag{88b}$$

$$\mathbb{E}(L_{3\delta}) = \frac{(1-\delta)\left(1 - (1-\delta p)^{\kappa_3}\right)}{\delta} \qquad \text{expected leakage through Mode 3 (eq. (52))} \tag{88c}$$

$$\mathbb{E}(L_{4\delta}) = \frac{(1 - \delta f_4)\,(1 - (1-\delta p)^{\kappa_4})}{\delta f_4} \qquad \text{expected leakage through Mode 4 (eq. (70))} \tag{88d}$$

Expressions for all associated variances, and full distributions, are provided in Section 2.

Cycle-level: In the case detection is assumed perfect or imperfect $(0 < \delta \leq 1)$

$$\mathbb{E}(Z^{(\text{CSP2})}) = \mathbb{E}(M_1) + \frac{\mathbb{E}(M_2) + \mathbb{E}(M_4)}{1 - (1-\delta p)^{\kappa_4}} \qquad \text{expected number in CSP-2 (eq.(72))} \tag{89a}$$

$$\mathbb{E}(Z^{(\text{CSP3})}) = \mathbb{E}(M_1) + \frac{\mathbb{E}(M_2) + \mathbb{E}(M_3) + \mathbb{E}(M_4)(1-\delta p)^{\kappa_3}}{1 - (1-\delta p)^{\kappa_3+\kappa_4}} \qquad \text{expected number in CSP-3 (eq.(76))} \tag{89b}$$

Setting $\delta = 1$ for the case of perfect detection and $0 < \delta < 1$ otherwise (imperfect detection):

- For the expected number of arrivals in a cycle through CSP-2 and CSP-3, expressions for $\mathbb{E}(A_{j\delta})$ (results (83) or (86)) can be substituted for $\mathbb{E}(M_j)$ $(j = 1, 2, 3, 4)$ in (89a-b)
- For the expected number of inspections in a cycle through CSP-2 and CSP-3, expressions for $\mathbb{E}(I_{j\delta})$ (results (84) or (87)) can be substituted for $\mathbb{E}(M_j)$ $(j = 1, 2, 3, 4)$ in (89a-b)
- For expected leakage in a cycle through CSP-2 and CSP-3, expressions for $\mathbb{E}(L_{j\delta})$ (results (85) or (88)) can be substituted for $\mathbb{E}(M_j)$ $(j = 1, 2, 3, 4)$ in (89a-b).

Expressions for all associated distributions, from which variances are easily derived, are provided in the Section 3.2.

5 Conclusion

In this paper, we formulate simple algebraic expressions for all distributions, expected values and variances for the number of arrivals, the number of inspections and the leakage, with inspection sensitivity explicitly included — for each mode and for a full cycle of CSP-2 and CSP-3. The inclusion of inspection sensitivity is highly relevant to Australian biosecurity systems because the inspection process implemented at the border is, typically, imperfect. Results provide a simple and accurate means of determining expected values and variances relevant for all prevalence levels — including low prevalence. They explicitly express how system controls in each mode influence biosecurity risk, and thereby contribute an easily accessible and powerful guide for CSP design and analysis.

As discussed in [1], there are numerous natural extensions to the analysis presented here: a distribution for prevalence could be added to incorporate the nature of uncertainty surrounding this unknown value; clearance numbers, monitoring fractions and inspection-effort for an acceptable volume of leakage could be determined under a variety of conditions; and inference could be incorporated to update the risk associated with particular importers or pathways (see also [2]). Further, economic considerations could be integrated to design the CSP system by, for example, incorporating a trade-off between costs and associated risks (see, for example, [8, 15]). These extensions are straightforward using the formulation provided.

Biosecurity measures and 'optimal' system design, using concepts such as AOQ and AOQL (average outgoing quality proposed in [4]) and the potential effect of inspection sensitivity on system design are not explored in this work — although aspects were considered in [1]. We suggest, however, that inspection sensitivity could alter CSP design decisions in fundamental ways (see [1]), and the inclusion of system and parameter uncertainty could have similar effects. A further aspect relevant to effective CSP design is the probability that leakage is prevented, and the volume of leakage given that it occurs. Relevant formulations are straightforward to determine from the distributions provided in this work (as in [1]).

Low contamination prevalence is typical of many pathways for arriving or departing consignments of goods at the border. Future work is planned to consider the performance of CSP-1, CSP-2 and CSP-3 biosecurity systems in these low-prevalence settings; the effect of variation in prevalence across a collection of arriving consignments; and the inclusion of the imperfect detection process associated with group-sampling techniques, which are commonly applied at the border to reduce costs.

Results from this paper are intended to complement those formulated in [1], thereby extending the relevance of that work to a broad range of CSP systems implemented at the Australian border. This paper does not propose or assess alternative CSP design principles or optimal strategies — it develops statistical results appropriate for that assessment. Results provide a collection of efficient and practical tools that have not previously been available, but which are purpose-built to support current and alternative Australian biosecurity systems. The statistical approach chosen is more powerful than standard simulation methods for many applications, while remaining simple, fast, accurate and easily accessible.

Acknowledgements

I would like to acknowledge that, following the completion of [1], Phil Tennant and Tony Arthur asked if I had considered CSP-3. That comment motivated this work. And I would like to thank Greg Hood and Marco Kienzle for providing mode-level and cycle-level simulation results for comparison.

References

[1] B. Barnes, G. Hood, and F. Giannini. Analytical distributions and stastistics for managing import pathway risk using the continuous sampling plan CSP-1. ABARES Working Paper, 2022.

[2] B. Barnes, M. Parsa, F. Giannini, and D. Ramsey. Analytical Bayesian approach for the design of surveillance and control programs to assess pest-eradication success. *Theoretical Population Biology*, 2022. (Under review).

[3] G. Decrouez and A. Robinson. Bias-corrected estimation in continuous sampling plans. *Risk Analysis*, 38(1), 2018.

[4] H.F. Dodge. A sampling inspection plan for continuous production. *The Annals of Mathematical Statistics*, 14(3):264–279, 1943.

[5] H.F. Dodge and M. N. Torrey. Additional continuous sampling inspection regimes. *Industrial Quality Control*, 7(5):1–12, 1951.

[6] W. Feller. *An introduction to probability theory and its applications: Volume I*. John Wiley and Sons, Inc., New York, 2nd edition, 1958.

[7] D. T. Ghosh. The continuous sampling plan that minimises the amount of inspection. *Indian Journal of Statistics, Series B*, 50(3):412–427, 1988.

[8] A. Hald. The compound hypergeometric distribution and a system of single sampling inspection plans based on prior distributions and costs. *Technometrics*, 2(3):275–340, 1960.

[9] S. Hester, A. Rossiter, A. Robinson, J. Sibley, B. Woolcott, C. Aston, and Hanea A. CBIS/CSP sensitivity: incorporating pre-border analysis. CEBRA Project 170608, Report, 2020.

[10] M. Hoffmann and A. Robinson. Reporting performance under a CSP sampling design. CEBRA Project 1305b, Report, 2016.

[11] G. J. Lieberman. A note on Dodge's continuous inspection plan. *The Annals of Mathematical Statistics*, 24(3):480–484, 1953.

[12] G. J. Lieberman and H. Solomon. Multi-level continuous sampling plans. *The Annals of Mathematical Statistics*, 26:686, 1955.

[13] L. M. McShane and B. W. Turnbull. Probability limits on outgoing quality for continuous sampling plans. *Technometrics*, 33(4):393–404, 1991.

[14] A. Robinson, J. Bell, B. Woolcott, and E. Perotti. AQIS quarantine operations risk return: Imported plant-product pathways.

[15] A. Rossiter and S. Hester. Designing biosecurity inspection regimes to account for stakeholder incentives: An inspection game approach. *Economic Record*, 93(301):277–301, 2017.

Appendix

A Previously published supporting results

We provide a distribution (and its associated probability mass, expected value and variance) for the inspection-number, in a sequence of at most κ inspections, in which contamination is first detected — conditional on detection within this sequence of inspections. Full details of the formulation are given in [1].

Let H denote a random variable for the inspection-number, and let κ denote the sequence length, with p and $\mathbb{D}_\kappa$ as defined in the main text. The associated conditional probability mass and generating function for the distribution are, respectively,

$$\mathbb{P}(H=h|\mathbb{D}_\kappa) = \frac{p(1-p)^{h-1}}{1-(1-p)^\kappa} \qquad \Phi_{H|\mathbb{D}_\kappa}(s) = \frac{ps\,(1-((1-p)s)^\kappa)}{(1-(1-p)^\kappa)\,(1-(1-p)s)} \tag{A.1}$$

with expected value and variance

$$\mathbb{E}(H|\mathbb{D}_\kappa) = \frac{1-(1-p)^\kappa\,(1+\kappa p)}{p\left(1-(1-p)^\kappa\right)} \tag{A.2}$$

$$\mathbb{V}\text{ar}(H|\mathbb{D}_\kappa) = \frac{(1-p)\left(1+(1-p)^{2\kappa}\right)-(1-p)^\kappa\left((\kappa p)^2+2(1-p)\right)}{\left(p\left(1-(1-p)^\kappa\right)\right)^2}. \tag{A.3}$$

The expected value agrees with that provided in [4, 5].

B Comparison with CSP results reported in Dodge and Torrey (1951)

The foundational work for CSP-2 and CSP-3 is published by Dodge and Torrey (1951) [5]. We compare results in this paper with the expected values assuming perfect detection that are reported in that work. Dodge and Torrey [5] do not provide expressions for all expected values, and they do not consider full distributions and variance, or incorporate imperfect detection in their results.

For CSP-2, they assume two sampling modes — Modes 1, 2 and 4 of the main text — and they provide algebraic expressions for the expected number of inspections in a full cycle of their CSP-2 system. For their results they assume that the monitoring fraction is the same for both sampling modes (i.e., $f = f_2 = f_4$).

From Section 3 of the main text, a general expression for the expected number of arrivals, inspections or leakage for the CSP-2 model can be expressed

$$\mathbb{E}(Z) = \mathbb{E}(M_1) + \frac{\mathbb{E}(M_2)+\mathbb{E}(M_4)}{1-\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})}. \tag{B.1}$$

The total number of inspections in sampling mode (Modes 2 and 4), over a full cycle of CSP-2 (as implemented in [5]) can then be determined using results (84) of the main text,

$$\mathbb{E}(M_1) = \mathbb{E}(I_1) = \frac{1-(1-p)^{\kappa_1}}{p(1-p)^{\kappa_1}}, \qquad \mathbb{E}(M_2) = \mathbb{E}(I_2) = \frac{1}{p}, \qquad \mathbb{E}(M_4) = \mathbb{E}(I_4) = \frac{1-(1-p)^{\kappa_4}}{p}.$$

Substituting into (B.1), it then follows that the expected number of inspections in a full cycle, with all modes combined and assuming perfect detection, is given by

$$\begin{aligned}
\mathbb{E}(I) = \mathbb{E}(I_1) + \frac{\mathbb{E}(I_2)+\mathbb{E}(I_4)}{1-\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})} &= \frac{1-(1-p)^{\kappa_1}}{p(1-p)^{\kappa_1}} + \frac{\frac{1}{p} + \frac{1-(1-p)^{\kappa_4}}{p}}{1-(1-p)^{\kappa_4}} \\
&= \frac{1-(1-p)^{\kappa_1}}{p(1-p)^{\kappa_1}} + \frac{2-(1-p)^{\kappa_4}}{p\left(1-(1-p)^{\kappa_4}\right)} \\
&= \frac{1}{p}\left(\frac{1}{(1-p)^{\kappa_1}} + \frac{1}{1-(1-p)^{\kappa}}\right),
\end{aligned}$$

where, in the first two lines, the first term is the expected number of inspections in Mode 1, through which there is at most one pass, and the second term is for the expected number of inspections in the two sampling modes combined, through which there may be numerous passes.

This expected value takes all possible transitions between different modes of CSP-2 into account, over one full cycle (before a return to Mode 1) — and our result agrees with the expected value provided in [5]. Our expression above has been determined from the distributions we formulated in the main text, but reduces to Eqs. (9) and (10) in [5] — noting that the expected number of inspections in Mode 1 can be expressed

$$\begin{aligned}
\mathbb{E}(I_1) &= \frac{1-(1-p)^{\kappa_1}}{(1-p)^{\kappa_1}}\left(\frac{1}{p} - \frac{\kappa_1(1-p)^{\kappa_1}}{1-(1-p)^{\kappa_1}}\right) + \kappa_1 \\
&= \frac{1-(1-p)^{\kappa_1} - \kappa_1 p(1-p)^{\kappa_1}}{p(1-p)^{\kappa_1}} + \kappa_1 \\
&= \frac{1-(1-p)^{\kappa_1}}{p(1-p)^{\kappa_1}} - \kappa_1 + \kappa_1 \\
&= \frac{1-(1-p)^{\kappa_1}}{p(1-p)^{\kappa_1}},
\end{aligned} \tag{B.2}$$

— where the first expression appears in [5], and the latter in result (84) of the main text.

Dodge and Torrey [5] also provide an an expression for AOQ associated with the CSP-3 model discussed in the main text (eq. 12 in [5]). Our algebraic simplification of the expression (not provided), as well as our numerical results and simulations, do not agree exactly with their expression — although they are close. However, result (76) in the main text does agree, exactly, with simulated results, as do each of the component distributions (Modes 1, 2, 3 and 4). We suggest that there is a 'typo' in their expression (eq. 12 in [5]) because all expected-values that they provide, as well as their expressions for the CSP-2 system, agree exactly with those derived using our expressions.

C Distribution for the number of arrivals, inspections and leakage in a cycle

We formulate full distributions for a cycle of CSP-2 and CSP-3, with that for CSP-1 provided in [1].

Expected values and variances are then straightforward to derive from these probability generating functions (C.3)–(C.5) (see below) in the usual way — that is (see standard statistical texts — for example, [6])

$$\mathbb{E}(Z) = \Phi'_Z(s)\Big|_{s=1}, \qquad \mathbb{V}\mathrm{ar}(Z) = \Phi''_Z(s)\Big|_{s=1} + \mathbb{E}(Z) - (\mathbb{E}(Z))^2, \tag{C.1}$$

where $\Phi'_Z(s)$ denotes the derivative with respect to s, which is evaluated at $s = 1$.

For reasons of parsimony we simplify the notation and set

$$\begin{aligned}
\Psi_{M_3a} &= \Phi_{M_3|\mathbb{D}_{\kappa_3}}(s)\,\mathbb{P}(\mathbb{D}_{\kappa_3}) \\
\Psi_{M_3b} &= \Phi_{M_3|\overline{\mathbb{D}}_{\kappa_3}}(s)\,\mathbb{P}(\overline{\mathbb{D}}_{\kappa_3}) \\
\Psi_{M_4a} &= \Phi_{M_4|\mathbb{D}_{\kappa_4}}(s)\,\mathbb{P}(\mathbb{D}_{\kappa_4}) \\
\Psi_{M_4b} &= \Phi_{M_4|\overline{\mathbb{D}}_{\kappa_4}}(s)\,\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4}).
\end{aligned} \tag{C.2}$$

CSP-2

CSP-2, as described in [5], excludes Mode 3 from the CSP-3 system (see Figure D.3). It follows that, for a single cycle of CSP-2 there is exactly one pass through Mode 1. There is at least one pass through Modes 2 and 4, and there may be many passes through both Modes 2 and 4. We construct a generating function for the 'generic' random variable Z (number of arrivals, inspections or leakage), for a full cycle, simple explicit expressions for the components of which have been formulated in the main text (Section 3.2):

$$\begin{aligned}
\Phi_Z(s) = \Phi_{M_1}(s)\Phi_{M_2}(s)\Big(&\mathbb{P}(\mathbb{D}_{\kappa_4})\Phi_{M_4|\mathbb{D}_{\kappa_4}}(s) + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\Phi_{M_4|\overline{\mathbb{D}}_{\kappa_4}}(s) \\
&\times\Phi_{M_2}(s)\Big(\mathbb{P}(\mathbb{D}_{\kappa_4})\Phi_{M_4|\mathbb{D}_{\kappa_4}}(s) + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\Phi_{M_4|\overline{\mathbb{D}}_{\kappa_4}}(s)\Phi_{M_2}(s)\,(\ldots)\Big)\Big) \\
= \Phi_{M_1}(s)\Big(&\Phi_{M_2}(s)\Psi_{4a} + \Phi^2_{M_2}(s)\Psi_{4b}\Psi_{4a} + \Phi^3_{M_2}(s)\Psi^2_{4b}\Psi_{4a} + \Phi^4_{M_2}(s)\Psi^3_{4b}\Psi_{4a} + \cdots\Big) \\
= \Phi_{M_1}(s)&\Phi_{M_2}(s)\Psi_{4a}\Big(1 + \Phi_{M_2}(s)\Psi_{4b} + (\Phi_{M_2}(s)\Psi_{4b})^2 + \ldots\Big) \\
= \Phi_{M_1}(s)&\left(\frac{\Phi_{M_2}(s)\Psi_{4a}}{1 - \Phi_{M_2}(s)\Psi_{4b}}\right).
\end{aligned} \tag{C.3}$$

Note that, applying (C.1), the expected value is

$$\begin{aligned}
\mathbb{E}(Z) &= \left[\Phi'_{M_1}(s)\left(\frac{\Phi_{M_2}(s)\Psi_{4a}}{1 - \Phi_{M_2}(s)\Psi_{4b}}\right)\right]_{s=1} \\
&+ \left[\Phi_{M_1}(s)\left(\frac{\left(\Phi'_{M_2}(s)\Psi_{4a} + \Phi_{M_2}(s)\Psi'_{4a}\right)\left(1 - \Phi_{M_2}(s)\Psi_{4b}\right) + \left(\Phi_{M_2}(s)\Psi_{4a}\right)\left(\Phi'_{M_2}(s)\Psi_{4b} + \Phi_{M_2}(s)\Psi'_{4b}\right)}{\left(1 - \Phi_{M_2}(s)\Psi_{4b}\right)^2}\right)\right]_{s=1} \\
&= \mathbb{E}(M_1)\frac{\mathbb{P}(\mathbb{D}_{\kappa_4})}{1 - \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})} + \frac{\left(\mathbb{E}(M_2) + \mathbb{E}(M_4|\mathbb{D}_{\kappa_4})\right)\mathbb{P}(\mathbb{D}_{\kappa_4})\left(1 - \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\right) + \mathbb{P}(\mathbb{D}_{\kappa_4})\left(\mathbb{E}(M_2) + \mathbb{E}(M_4|\overline{\mathbb{D}}_{\kappa_4})\right)\mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})}{\left(1 - \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\right)^2} \\
&= \mathbb{E}(M_1) + \frac{\mathbb{E}(M_2) + \mathbb{E}(M_4)}{1 - \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})}
\end{aligned} \tag{C.4}$$

— which agrees with result (72) (and (B.1)).

CSP-3

For a full cycle of CSP-3, as described in [5] (see Figure 1), there is exactly one pass through Mode 1. There is at least one pass through Modes 2 and 3; there may be no passes through Mode 4; or there may be many passes through all of Modes 2, 3 and 4. We construct a generating function for 'generic' random variable Z (number of arrivals, inspections or leakage), for a full cycle, simple explicit expressions for the components of which have been formulated in the main text (Section 3.2):

$$\begin{aligned}
\Phi_Z(s) &= \Phi_{M_1}(s)\Phi_{M_2}(s)\Big(\mathbb{P}(\mathbb{D}_{\kappa_3})\Phi_{M_3|\mathbb{D}_{\kappa_3}}(s) + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\Phi_{M_3|\overline{\mathbb{D}}_{\kappa_3}}(s)\Big(\mathbb{P}(\mathbb{D}_{\kappa_4})\Phi_{M_4|\mathbb{D}_{\kappa_4}}(s) + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_4})\Phi_{M_4|\overline{\mathbb{D}}_{\kappa_4}}(s) \\
&\qquad \times\Phi_{M_2}(s)\Big(\mathbb{P}(\mathbb{D}_{\kappa_3})\Phi_{M_3|\mathbb{D}_{\kappa_3}}(s) + \mathbb{P}(\overline{\mathbb{D}}_{\kappa_3})\Phi_{M_3|\overline{\mathbb{D}}_{\kappa_3}}(s)\Big(\mathbb{P}(\mathbb{D}_{\kappa_4})\Phi_{M_4|\mathbb{D}_{\kappa_4}}(s) + \ldots\Big)\Big)\Big)\Big) \\
&= \Phi_{M_1}(s)\Big(\Phi_{M_2}(s)\Psi_{3a} + \Phi_{M_2}(s)\Psi_{3b}\Psi_{4a} + \Phi^2_{M_2}(s)\Psi_{3b}\Psi_{4b}\Psi_{3a} + \Phi^2_{M_2}(s)\Psi^2_{3b}\Psi_{4b}\Psi_{4a} \\
&\qquad +\Phi^3_{M_2}(s)\Psi^2_{3b}\Psi^2_{4b}\Psi_{3a} + \Phi^3_{M_2}(s)\Psi^3_{3b}\Psi^2_{4b}\Psi_{4a} + \Phi^4_{M_2}(s)\Psi^3_{3b}\Psi^3_{4b}\Psi_{3a} + \ldots\Big) \\
&= \Phi_{M_1}(s)\Big(\Phi_{M_2}(s)\Psi_{3a}\Big(1 + \Phi_{M_2}(s)\Psi_{3b}\Psi_{4b} + (\Phi_{M_2}(s)\Psi_{3b}\Psi_{4b})^2 + (\Phi_{M_2}(s)\Psi_{3b}\Psi_{4b})^3 + \cdots\Big) \\
&\qquad +\Phi_{M_2}(s)\Psi_{3b}\Psi_{4a}\Big(1 + \Phi_{M_2}(s)\Psi_{3b}\Psi_{4b} + (\Phi_{M_2}(s)\Psi_{3b}\Psi_{4b})^2 + (\Phi_{M_2}(s)\Psi_{3b}\Psi_{4b})^3 + \cdots\Big)\Big) \\
&= \Phi_{M_1}(s)\left(\frac{\Phi_{M_2}(s)\Psi_{3a} + \Phi_{M_2}(s)\Psi_{3b}\Psi_{4a}}{1 - \Phi_{M_2}(s)\Psi_{3b}\Psi_{4b}}\right). \qquad \text{(C.5)}
\end{aligned}$$

The expected value and variance can then be found in the usual way (see (C.1)).

D Schematic diagrams for distinct continuous sampling plans

We provide schematic diagrams for the three distinct sampling plans considered.

A schematic diagram of CSP-1, as originally proposed in [4], is provided in Figures D.1 and D.2. The former provides details of the inner processes in each of the two modes, and the latter is in the same format as alternative schemes and provided for purposes of comparison. In CSP-1 there is a single sampling mode — Mode 2.

A schematic diagram of CSP-2, as originally proposed in [5], is provided in Figure D.3. Relative to CSP-1, this scheme has an extra sampling mode — Mode 4. Thus it has two distinct sampling modes — Modes 2 and 4. Details for the inner processes of Modes 1 and 2 are identical to those of CSP-1 (Figure D.1).

A schematic diagram of CSP-3, as originally proposed in [5], is provided in Figure 1 of the main text and is replicated here (Figure D.4). Relative to CSP-2, this scheme has the same two sampling modes with an extra 100%–inspection mode included — Mode 3. Details for the inner processes of Modes 1 and 2 are identical to those of CSP-1 (Figure D.1).

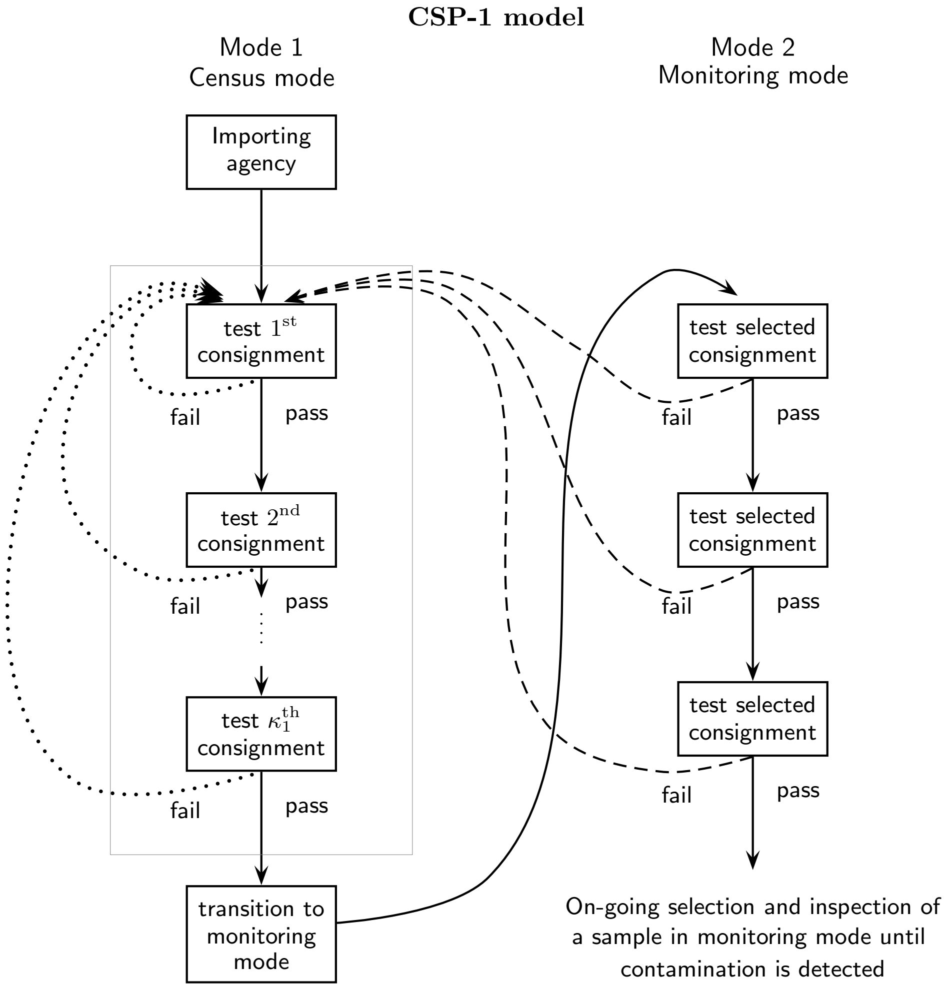

Figure D.1: Schematic diagram of the CSP-1 system originally proposed in [4]. In Mode 1 (Census Mode, left-hand-column), where all arriving consignments are inspected, a detection ('fail') initiates a new block (grey box) of inspections (dotted curves); a sequence of κ_1 inspections with no detections leads to a transition to Mode 2 or Monitoring Mode (solid curve); in Mode 2 (right-hand-column), where a monitoring-fraction f_2 of arriving consignments are selected and inspected, a detection ('fail') returns the importer to Mode 1 or Census Mode (dashed curves). A full CSP-1 cycle is complete when an importer who started in Mode 1 (Census Mode), transitions to Mode 2 (Monitoring Mode) and is subsequently returned to Mode 1. Mode 1 (Census Mode) is also referred to as 'Enhanced Inspection Mode' [14].

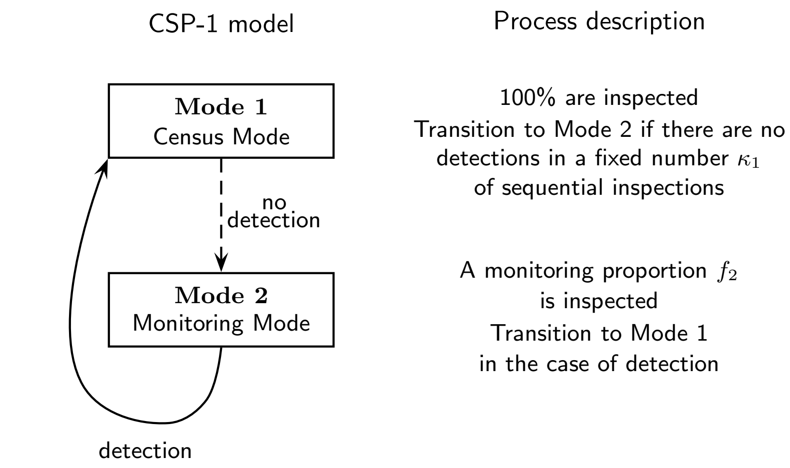

Figure D.2: Schematic diagram of the CSP-1 system originally proposed in [4]. The inspection process begins in Mode 1 (Census Mode) and a full cycle is complete with a return to Mode 1 (Census Mode) — outer solid curve on the left-hand-side. Mode 1 (Census Mode) is also referred to as 'Enhanced Inspection Mode' by CEBRA [14].

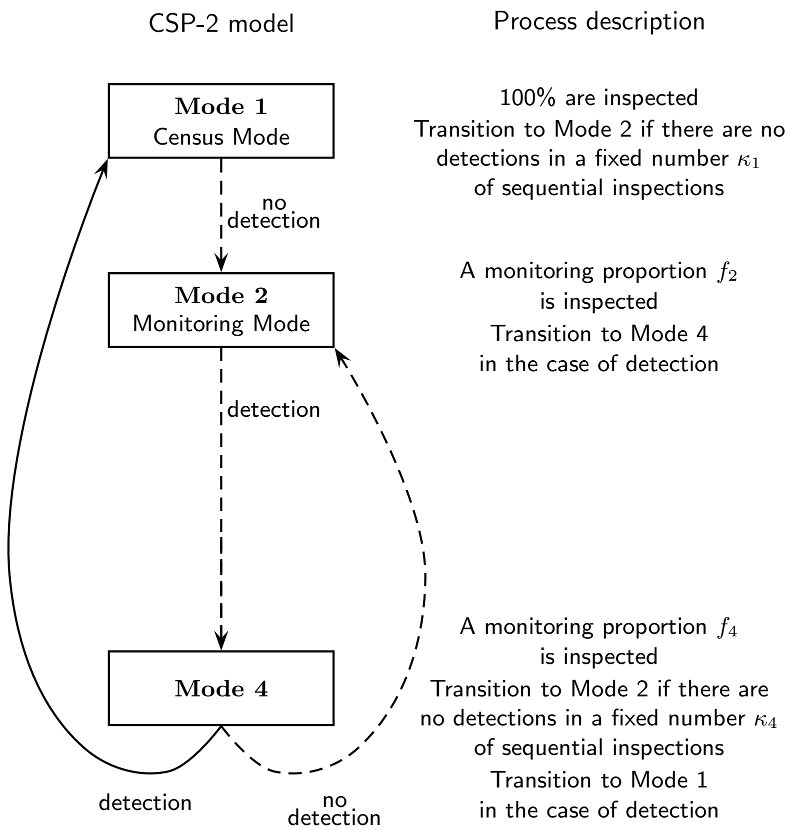

Figure D.3: Schematic diagram of the CSP-2 system originally proposed in [5]. The inspection process begins in Mode 1 (Census Mode) and a full cycle is complete with a return to Mode 1 (Census Mode) — outer solid curve on the left-hand-side. Mode 1 (Census Mode) is also referred to as 'Enhanced Inspection Mode' by CEBRA [14], and Mode 4 is also referred to as 'Alert' Mode by CEBRA [10].

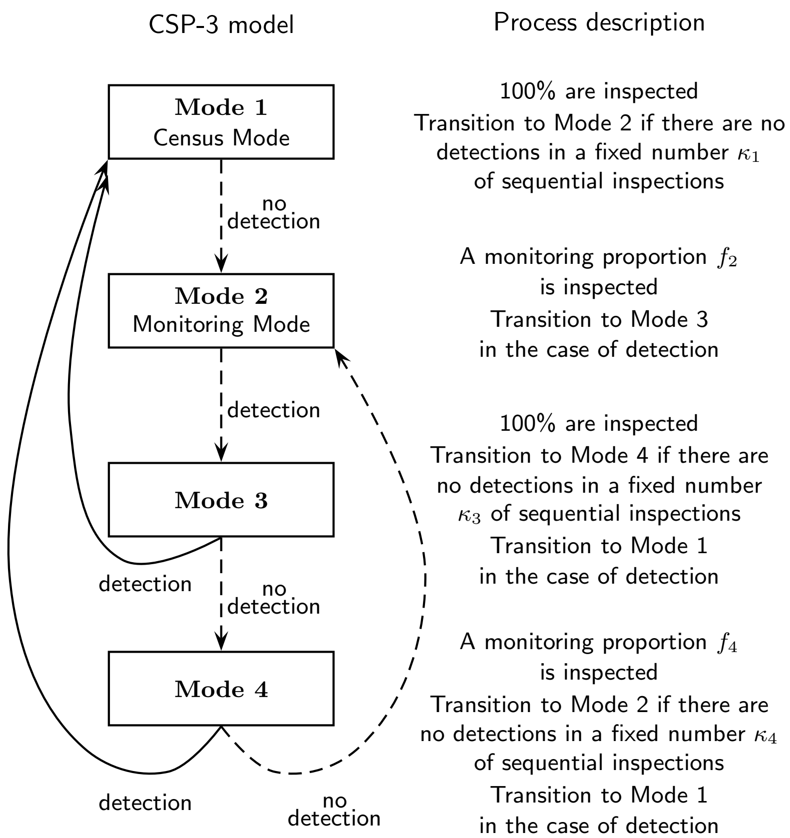

Figure D.4: Schematic diagram of the CSP-3 system originally proposed in [5]. The inspection process begins in Mode 1 (Census Mode) and a full cycle is complete with a return to Mode 1 (Census Mode) — outer solid curves on the left-hand-side. Mode 1 (Census Mode) is also referred to as 'Enhanced Inspection Mode' by CEBRA [14], Mode 3 is also referred to as 'Limbo' Mode by CEBRA [10] and as 'Tight' Mode in DAFF (Animal Biosecurity), and Mode 4 is also referred to as 'Alert' Mode by CEBRA [10].